1984-03

Numerical computation of the scattering coefficients of an inductive strip in a fin-line

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http://hdl.handle.net/10945/26654

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THESIS

NUMERICAL COMPUTATION OF THE SCATTERING
COEFFICIENTS OF AN INDUCTIVE
STRIP IN A FIN-LINE

by

John C. Deal

March 1984

Thesis Advisor: J. B. Knorr

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Numerical Computation of the Scattering Coefficients of an Inductive Strip in a Fin-Line		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; March 1984
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John C. Deal		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		12. REPORT DATE March 1984
		13. NUMBER OF PAGES 178
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Special Distribution

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Microwave
Millimeter Wave
Integrated Circuits
Inductive Strips
Septums
Discontinuities
Fin-Line

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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S/N 0102-LF-014-6601

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Numerical Computation of the Scattering Coefficients of an Inductive Strip in a Fin-Line

by

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Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
March 1984

ABSTRACT

Fin-line is well recognized as a viable transmission structure for millimeter waves. The theoretical analysis of fin-line has received considerable attention and has been discussed in the literature by numerous authors. The spectral domain method of analysis has been established as a powerful and versatile approach to computing the characteristics of fin-line derived structures. This thesis describes the application of the spectral domain technique to the solution of the problem of end coupled fin-line cavities. Galerkin's method is used to compute the odd and even mode resonant lengths which are then used to calculate the scattering coefficients of the inductive strip separating the two cavities. Results are compared with experimental measurements.

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ACKNOWLEDGMENTS

I would like to thank Professor Jeffery B. Knorr for his continued support, guidance, and most of all faith. I would also like to thank Dr. Yi-Chi Shih for his counsel and inspirational suggestions and Dr. H. M. Lee for his continued interest and provoking questions. Most importantly, I would like to thank my wife, Colleen, for her assistance in preparing the figures and graphs contained within and proofing the draft and final document. Her continued emotional support has made the completion of this research possible.

I. INTRODUCTION

The need and advantages, as well as difficulties, of going to millimeter waves (30-300 GHz) are recognized. As a consequence, numerous transmission media have been developed in an effort to take advantage of present technology and overcome the drawbacks. The objectives of these new designs are low attenuation, single mode operation, containment of spurious emissions, ease of production, and compatibility with integrated circuit technology. One such design which appears very attractive is the Fin-Line transmission structure [Refs. 1, 2].

The Spectral Domain technique of analyzing Fin-Line structures is well established in the research community [Refs. 3-11]. A review of these references shows that difficulties lie in the viable choice of basis functions to represent the fields within the Fin-line structure. Two papers of prime importance in the establishment of the spectral domain technique for analyzing Fin-Line structures are by Knorr and Shayda in July 1980 [Ref. 4] and Peter-Schmidt and Itoh in September 1980 [Ref. 5]. Both addressed the proper choice of basis functions and accurately determined the Fin-Line guide wavelength and characteristic impedance. What is important is that their methods and approach have set the standard in Fin-Line analysis.

The purpose of this thesis is to analyze the inductive discontinuity (strip) in a Fin-Line structure. This problem was first addressed in November 1981 by Knorr [Ref. 7]. Therefore, this thesis is a direct extention of Knorr's work. The analysis was accomplished in three phases. First, an extended set of basis functions were chosen and the results presented in Reference 7 were reproduced. This was the evaluation of the resonant length of a single resonant cavity. Second, another set of basis functions was chosen and the odd and even resonant lengths of two unilaterally coupled Fin-Line cavities were evaluated. Third, with the odd and even mode resonant lengths obtained and treating the inductive discontinuity as a lossless two-port network, the S-parameters were evaluated in terms of the odd and even resonant lengths. The numerical results were then compared with the experimental results obtained by Miller [Ref. 12]. This thesis will discuss both the theoretical and numerical aspects of the analysis.

II. SINGLE CAVITY RESONANT LENGTH AND EQUIVALENT REACTANCE OF A SHORTING SEPTUM

The following is broken down into two parts: theory and numerical analysis and results. The theory covers the method of moments, Galerkin's method, Fin-Line structure and selected basis set, implementation of Galerkin's method, and definition of inner product and equivalent reactance. The numerical analysis and results cover the search method, functional singularities, limits of summation, convergence, and comparison with other numerical and experimental results.

A. THEORY

Harrington in Reference 13 provides a full and complete discussion of the computation of fields by the method of moments. Briefly, if $L(f) = g$ represents an equation where L is a linear operator, g is a known function (or possibly unknown) and f is a function which represents the solution to be determined, allowing f to be represented by a series of basis functions f_n a solution may be arrived at in the following manner:

Given

$$L(f) = g \tag{1}$$

Let

$$f = \sum_n \alpha_n f_n \tag{2}$$

where the α_n's are to be determined. Substituting (2) into (1) yields

$$\sum_n \alpha_n L(f_n) = g$$

Multiplying by W_m,

$$\Sigma \alpha_n w_m L(f_n) = w_m g$$

where w_m is some weight function to be determined.

Taking the inner product, $<a,b>$, of both sides:

$$\Sigma \alpha_n <w_m, L(f_n)> = <w_m, g>$$

where the inner product is to be determined. Allowing $w_m = f_m$:

$$\Sigma \alpha_n <f_m, L(f_n)> = <f_m, g> .$$

This is known as Galerkin's Method.

In matrix algebra format, the above becomes:

$$[<f_i, L(f_j)>] \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} = [<f_i, g>]$$

where i=1 to m and j=1 to n.

The theoretical analysis of the single resonant cavity begins by continuing with Eq. (12), Reference 7, and making the assumption that $\mathcal{E}_z \approx 0$ leaving:

$$\langle G_{11}\, \mathcal{E}_x, \mathcal{E}_x \rangle = 0$$

where the script represent the Fourier transform of the x-directed electric field, and G_{11} is the dyadic Green's function obtained by the application of the Helmholtz Equation and Boundary Conditions to the Fin-Line geometry as depicted in Figure II-1. Here "a" is the width of the guide, "b" is the height of the guide, D is the dielectric thickness, h_1 and h_2 represent the fins location relative to the side walls of the guide and w is the gap width. ε_{r_1}, ε_{r_2}, and ε_{r_3} are the relative dielectric constants for regions 1, 2, and 3 respectively.

The following assumption is made in the transform (spectral) domain:

$$\mathcal{E}_x(\alpha_n,\xi_k) = \mathcal{E}_x^x(\alpha_n)\mathcal{E}_x^z(\xi_u)$$

where α_n and ξ_k are the transform variables with respect to x and z and are defined by Equations (4) and (5), Reference 7. Next step is the choice of basis functions.

The x component of the electric field is defined as:

$$e_x(x,z) = A \sum_{m=1}^{M} B_m \cos \frac{(2m-1)\pi z}{\ell}$$

for $|x| \leq w/2$, $|z| \leq \ell/2$.

The geometries for the fin-line resonant cavity and the z-dependence of the x-directed electric field, $e_x(x,z)$ are as depicted by Figure II-2.

In the transform (spectral) domain,

$$F\{e_x(x,z)\} = E_x^x(\alpha_n) \sum_{m=1}^{M} B_m E_{x_m}^z (\xi_k)$$

where

$$E_x^x(\alpha_n) = AW \frac{\sin \frac{1}{2}(\alpha_n D)(\frac{W}{D})}{\frac{1}{2}(\alpha_n D)(\frac{W}{D})}$$

and

$$E_{x_m}^z (\xi_k) = (-1)^m \frac{(2m-1)}{2}(\ell\pi) \frac{\cos(\xi_k D)(\ell/2D)}{(\xi_k D)^2(\frac{\ell}{2D})^2 - [\frac{(2m-1)\pi}{2}]^2} \quad .$$

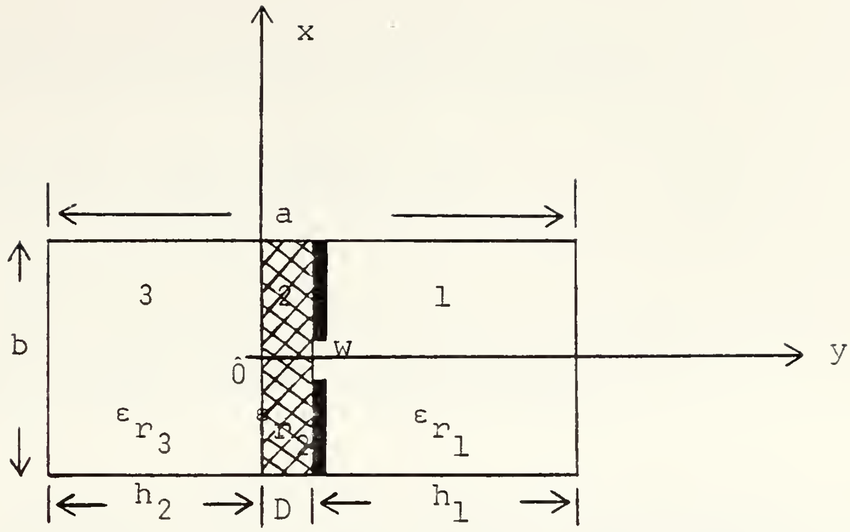


Figure II-1 Fin-Line Structure Geometry (end on view)

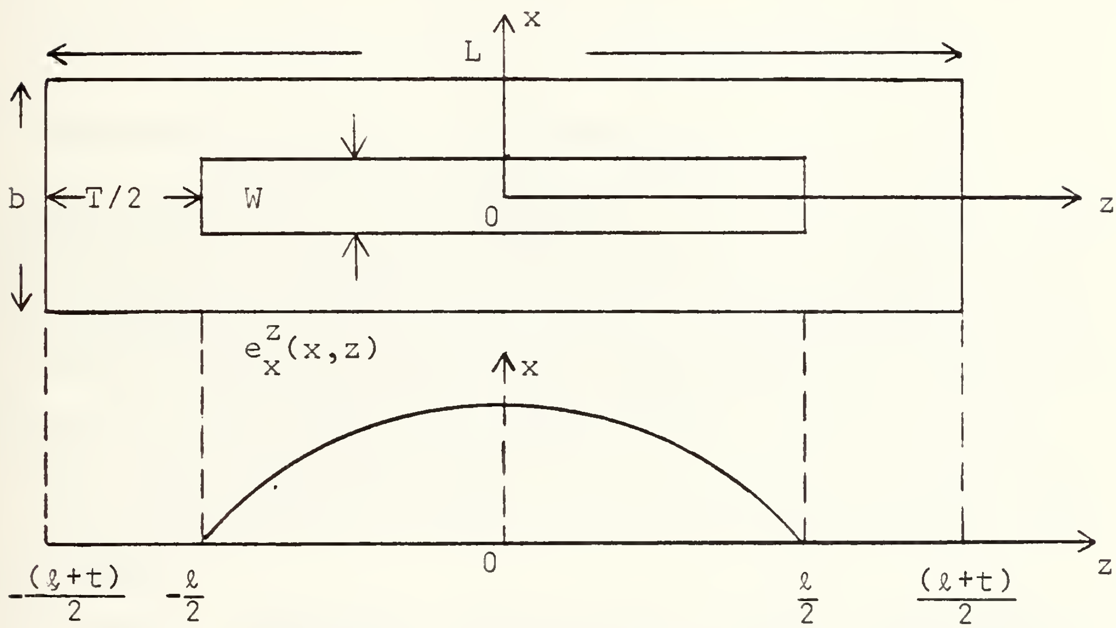


Figure II-2 Fin-Line Resonant Cavity Geometry (side view and z- dependence of the electric field)

Finally, applying the Galerkin's Method yields:

$$<B_j[E_x^x(\alpha_n)]^2 \; G_{11} \; E_{x_i}^z \; (\xi_k)E_{x_j}^z \; (\xi_k)> \quad = 0$$

$$[<[E_x^x(\alpha_n)]^2 \; G_{11} \; E_{x_i}^z \; (\xi_k)E_{x_j}^z \; (\xi_k)>] \; [\tilde{B}_j] \quad = \quad 0$$

where i = 1 to M and j = 1 to M. And the inner produce is defined as

$$\sum_{n=-\infty}^{\infty} \quad \sum_{n=-\infty}^{\infty} G_{11}(\alpha_n,\xi_k)[E_x^x(\alpha_n)]^2 \; E_{x_i}^z \; (\xi_k)E_{x_j}^z(\xi_k) \quad \text{[Ref. 13].}$$

For a complete derivation see Appendix A.

B. NUMERICAL ANALYSIS AND RESULTS

The problem is to find the value of the resonant length for a given Fin-Line geometry and frequency which causes the determinant of the matrix of inner profuct terms to equal zero, since the B coefficients cannot be zero except in the trivial case.

For ease in numerical calculations and for programming purposes all geometric parameters and frequencies are normalized as follows:

D/λ Wavelength (frequency) of interest normalized with respect to D, the dielectric thickness

h_1/D Fin location relative to the positive "y" side wall normalized with respect to D

h_2/D Fin location relative to the negative "y" side wall normalized with respect to D

b/D Guide height normalized with respect to D

t/D Inductive Strip width normalized with respect to D.

The resonant length normalized to the guide wavelength is defined as ℓ/λ' where λ', the guide wavelength, is numerically determined in the same manner as described in Reference 4. A simple bisectional search is used to determine the value of ℓ/λ' for which the determinant goes to zero. The residue is .0001, therefore, the uncertainty of ℓ/λ' is $\pm$ .0001. Increasing the size of the matrix (number of basis functions used) continually improves the result until a limit is reached where no further improvement is realized. This is depicted by the convergence test of Figure II-3.

Two other numerical considerations are singularities in the $E_x^x(\alpha_n)$ and $E_x^z(\xi_k)$ functions and the limits of summation over α_n and ξ_k. Both are derived and resolved in Appendix A. The documented program is found in Appendix G.

Numerical results for a single cavity (Figure II-2) are graphed and may be found in Appendix B. The figures of Appendix B are of the same form as Figures 7 through 12 of Reference 7, hence comparison is made with those results.

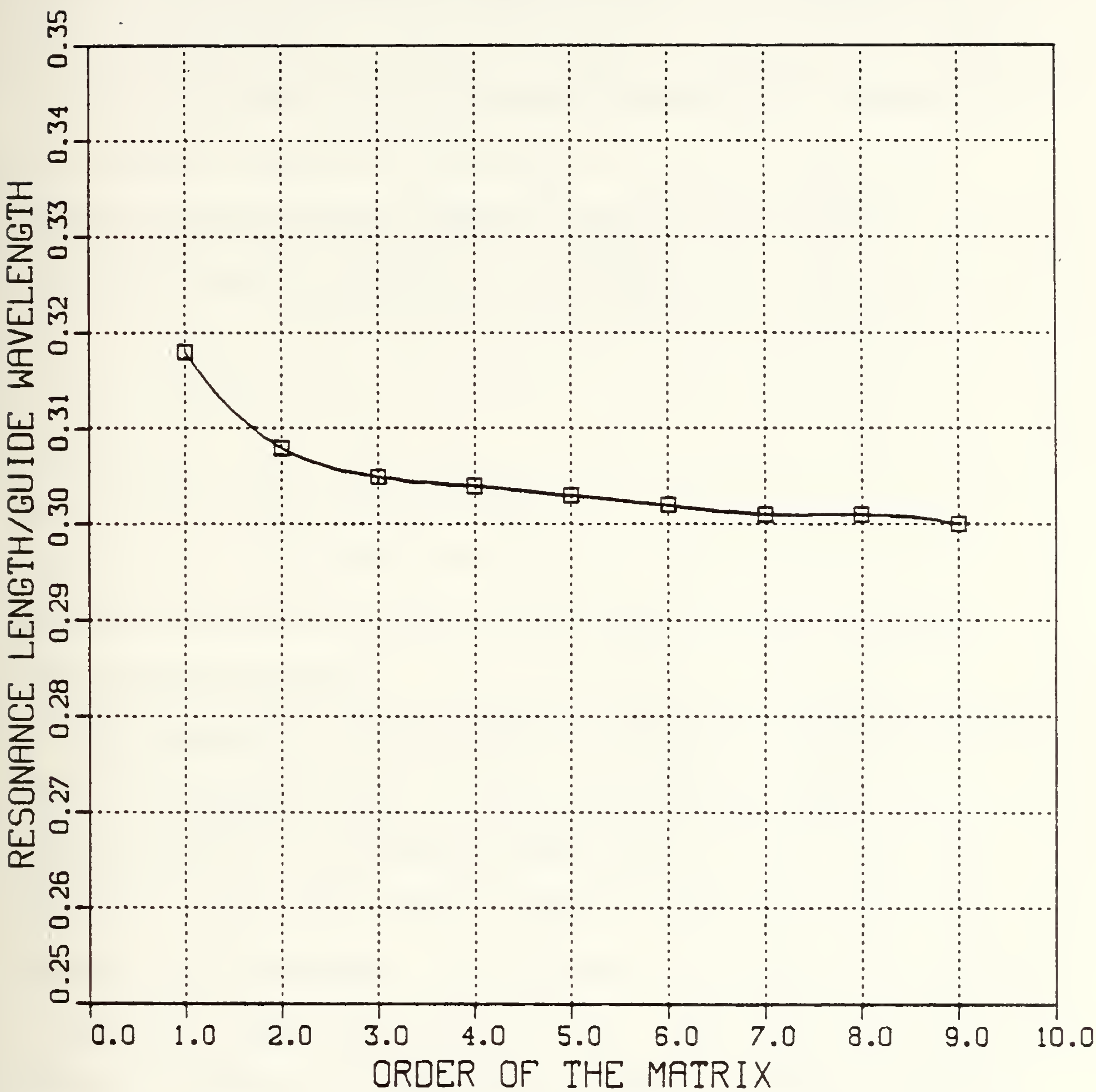


Figure II-3 Convergence Test for the Single Resonant Cavity for w/b=0.5, t/D=16.0, and f=12 GHz

The solid curves and circled crosses in these graphs are the numerical and experimental results respectively from Reference 7. The dashed curves are the numerical results obtained by the method of moments with the expanded set of cosine functions.

As can be seen, the difference between the experimental and numerical results of Reference 7 and the numerical results obtained herein ranges from 0% to 2%. The normalized septum reactance is defined as

$$x = \tan 2\pi\, \Delta\ell/\lambda'$$

where $\Delta\ell/\lambda' = \frac{1}{2}(\frac{1}{2} - \ell/\lambda')$, [Ref. 7]. Figure B-4 shows the variation of the normalized reactance and in turn, the normalized resonant length, with respect to the length of the inductive septum.

The dimension of the matrix was chosen to be 7x7 based upon a convergence test in which the matrix order was iterated until the resonant length was no more than 0.001 (typically less than 0.5%). Given a particular fin-line geometry and frequency, the solutions converged with matrix dimensions ranging from 5x5 to 8x8. The numerical calculations for Figures B-1 through B-4 were for WR(90) waveguide with centered fins for 8.0 to 12.0 GHz, normalized gap width w/b = 1.0, 0.5, and 0.1, dielectric thickness D=0.1 inch,

a = 0.9 inch, b = 0.4 inch, and for a dielectric constant, $\varepsilon_{r_2} = 1.0$. The numerical calculations for Figures B-5 through B-7 were for normalized frequencies $2a/\lambda = 1.2$ to 1.8 normalized gap width w/b = 1.0, 0.5, 0.2 and 0.1, b/a = 0.5, and D/a = 0.1, 0.05, and 0.1 respectively for $\varepsilon_{r_2} = 1.0$ and 2.2.

The conclusion based upon this phase of the analysis is that an expanded set of basis functions works extremely well and that the choice of the cosine basis functions to represent the z-variation of the x-directed electric field was proper and viable. This set the stage for phase two of the analysis.

III. COUPLED CAVITIES RESONANT LENGTHS FOR ODD AND EVEN MODES

The following is broken down into two parts, theory and numerical analysis and results. The theory will cover the geometry of the two coupled fin-line cavities, the selected basis set, odd and even mode field distribution, and implementation of Galerkin's Method. The numerical analysis and results will cover convergence and three tests of the results for correctness.

A. THEORY

The theoretical analysis of the two coupled cavities is an extension of the single cavity. The geometrical structure of the coupled cavities and field distributions of the selected basis functions for the z-dependence of the x-directed electric field E_x^z for both odd and even modes are described in Figure III-1. The end walls are perfect shorts and the inductive discontinuity is in the center of the structure. The solid lines of the field distributions represent even modes and the dashed lines represent odd modes.

As can be seen the choice of basis functions are both sine and cosine. The x and z dependence of the electric field is given by

$$e_x(x,z) = A \sum_{q=1}^{Q} B_q F_q(z), \quad |x| \leq w/2, \quad t/2 \leq |z| \leq \ell + t/2$$

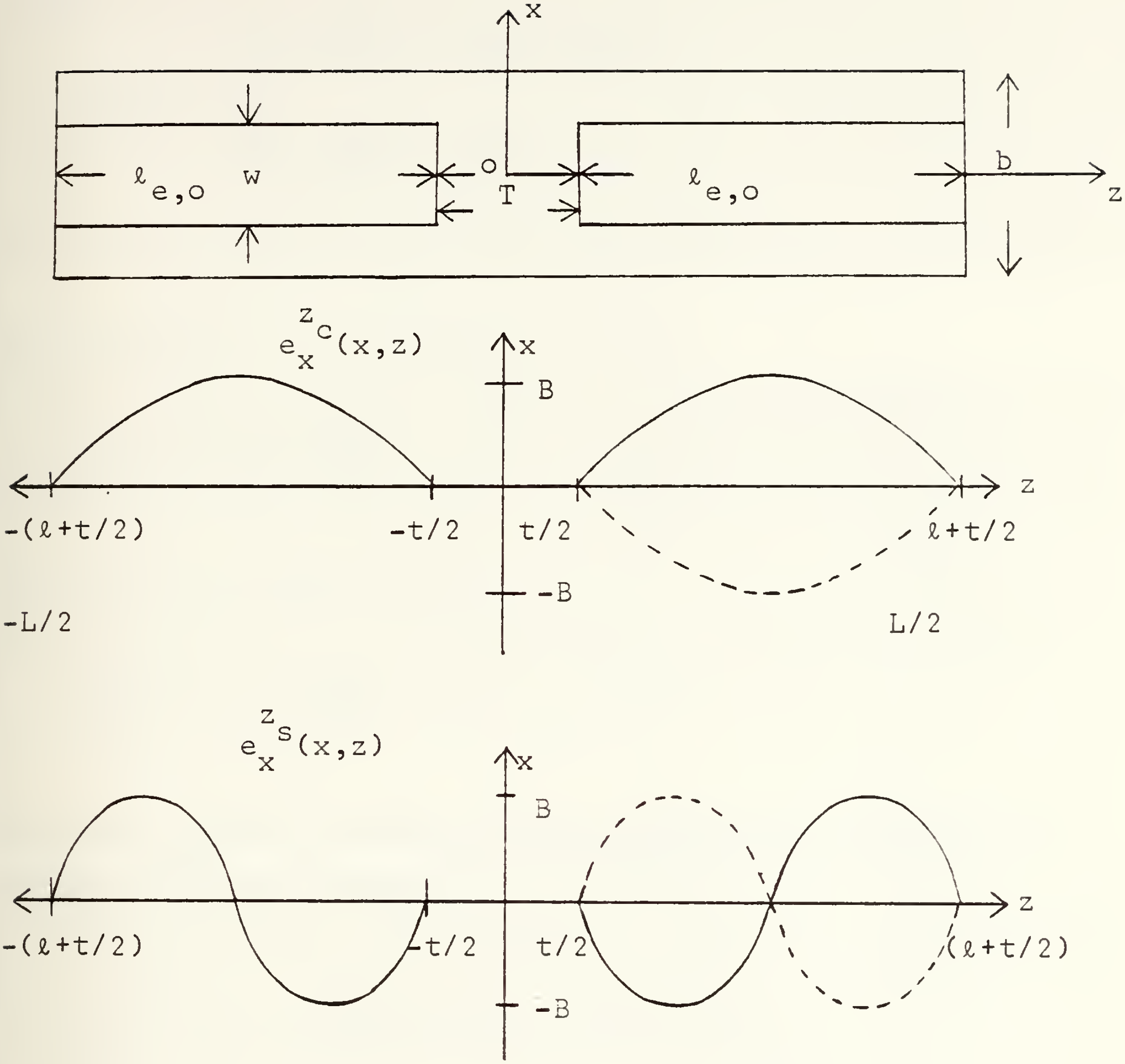


Figure III-1 Coupled Fin-Line Cavities and Basis Functions Field Distribution (side view)

where

$$F_q(z) = \begin{cases} \sin \frac{q\pi z}{\ell} \text{ , for } q \text{ even.} \\ \\ \cos \frac{q\pi z}{\ell} \text{ , for } q \text{ odd.} \end{cases}$$

For example, if q = 4, which is even,

$$F_4(z) = \sin \frac{4\pi z}{\ell}$$

and if q = 7, which is odd,

$$F_7(z) = \cos \frac{7\pi z}{\ell} \quad .$$

And the spatially shifted basis functions as depicted in Figure III-1 are defined

$$F_q(z) = \begin{cases} -\sin \frac{q\pi}{\ell} [z + (\frac{\ell+t}{2})], & z<0 \\ & q \text{ even} \\ \pm \sin \frac{q\pi}{\ell} [z - (\frac{\ell+t}{2})], & z>0 \\ & q \text{ even} \\ \cos \frac{q\pi}{\ell} [z + (\frac{\ell+t}{2})], & z<0 \\ & q \text{ odd} \\ \pm \cos \frac{q\pi}{\ell} [z - (\frac{\ell+t}{2})], & z>0 \\ & q \text{ odd} \end{cases}$$

where for ($\pm$) the sine function, plus (+) = odd mode and minus (-) = even mode, and where for ($\pm$) the cosine function, plus (+) = even mode and minus (-) = odd mode.

In the transform domain,

$$F\{e_x(x,z)\} = E_x^x(\alpha_n) \sum_{q=1}^{Q} B_q E_{x_q}^z(\xi_k)$$

where α_n and ξ_k are the x and z transform variables respectively as defined by Eq. (4) & (5), Reference 7.

For q even:

$$E_{x_q}^{z_s}(\xi_k) = \begin{cases} (-1)^q \left(\frac{q\pi}{2}\ell_e\right) \dfrac{\sin\theta}{\theta^2-\left(\frac{q\pi}{2}\right)^2} (2) \sin \xi_k\left(\dfrac{\ell_e+t}{2}\right) \\ \qquad \text{for Even Mode} \\ (-1)^q \left(\frac{q\pi}{2}\ell_o\right) \dfrac{\sin\theta}{\theta^2-\left(\frac{q\pi}{2}\right)^2} (2j) \cos \xi_k\left(\dfrac{\ell_e+t}{2}\right) \\ \qquad \text{for Odd Mode} \end{cases}$$

and for q odd:

$$E_{x_q}^{z_c}(\xi_k) = \begin{cases} (-1)^q \frac{q}{2}(\ell_e\pi) \dfrac{\cos\theta}{\theta^2-\left(\frac{q\pi}{2}\right)^2} (2)\cos\xi_k\left(\dfrac{\ell_e+t}{2}\right) \\ \qquad \text{for Even Mode} \\ (-1)^q \frac{q}{2}(\ell_o\pi) \dfrac{\cos\theta}{\theta^2-\left(\frac{q\pi}{2}\right)^2} (2j)\sin\xi_k\left(\dfrac{\ell_o+t}{2}\right) \\ \qquad \text{for Odd Mode} \end{cases}$$

where $\theta = (\xi_k \ell)/2$.

Finally applying Galerkin's Method yields:

$$[<[E_x^x(\alpha_n)^2\ G_{11} E_{x_i}^z(E_u) E_{x_j}^z(\xi_k)>]\ [\tilde{B}_j] = 0$$

where "i" and "j" = 1 to Q and $E_{x_i}^z$ and $E_{x_j}^z$ are sine or cosine functions depending on whether the index is odd or even.

The inner product is defined over α_n and ξ_k, as it was in the single cavity case. For a complete derivation see Appendix C.

B. NUMERICAL ANALYSIS AND RESULTS

The problem is to find the values of the odd and even mode resonant lengths of the coupled cavities for a given Fin-Line geometry and frequency which cause the determinant of the matrix of inner products to equal zero. The odd and even mode resonant lengths normalized to the guide wavelength are denoted by ℓ_o/λ' and ℓ_e/λ', respectively. As in the single cavity case, a bisectional search is used with a residue of .0001, therefore the values of the odd and even resonant lengths are determined to within $\pm$ .0001. Increasing the size of the matrix (number of basis functions used) continually improves the result until a numerical limit in computation is reached. That limit is a matrix of 12x12 dimension. However, the numerical results do converge with increasing order of the matrix. This is depicted by the

convergence test of Figure III-2. Therefore an order of 12 was used for all numerical results contained herein for the coupled cavities where w/b=1.0.

Three checks are used to determine the correctness of the numerical results. First, as the thickness of the inductive strip increases a point is reached where the two cavities become uncoupled and $\ell_e/\lambda' = \ell_o/\lambda'$. This is seen to occur in the numerical results of Appendix D for w/b = 1.0 where the Fin-Line gap equals the "b" dimension of the waveguide. As the thickness of the strip, normalized with respect to D, t/D (TOVD) approaches 16.0, ℓ_e/λ' (LE/LPR) equals ℓ_o/λ' (LO/LPR). Here D = 0.1 inch and D/λ (D/L) is the free space wavelength normalized with respect to D, the dielectric thickness, for 8.0 to 12.0 GHz in steps of 0.5 GHz. Second, as the thickness of the strip becomes very small ℓ_o/λ' approaches one half (0.5). Referring to Appendix D, this can be seen to occur. In fact, as t/D = 0.1, $\ell_o/\lambda' = 0.5$ for all frequencies from 8 to 12 GHz. The third check is noting that the odd mode resonant length is equal to the resonant length for the single cavity plus the equivalent electrical length of the inductive septum, as derived in the single cavity case. Graphically, this is depicted in Figure III-3.

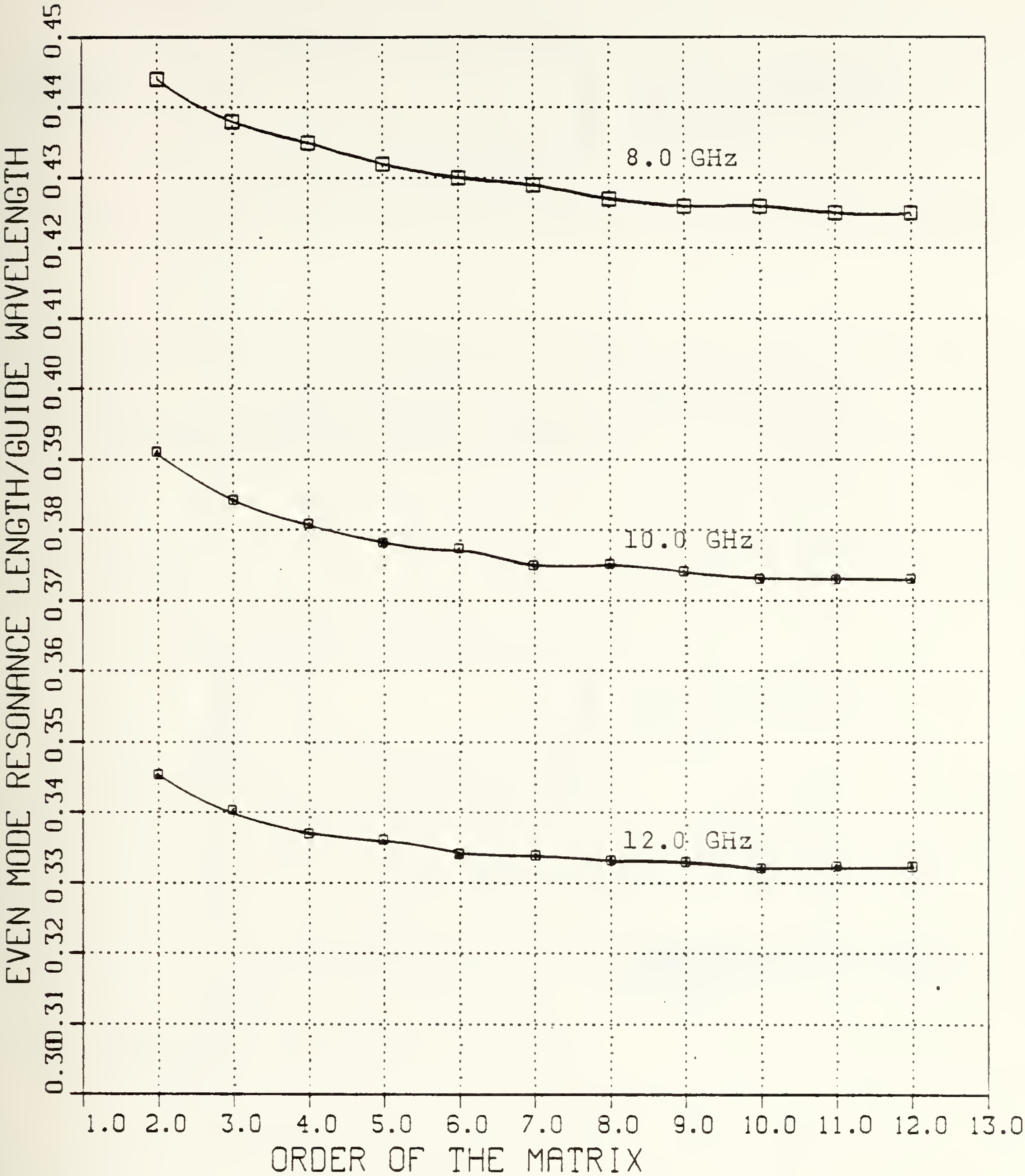


Figure III-2 Convergence Test for the Coupled Resonant Cavities for w/b=1.0, t/D=1.0 and f=8.0, 10.0, and 12.0 GHz

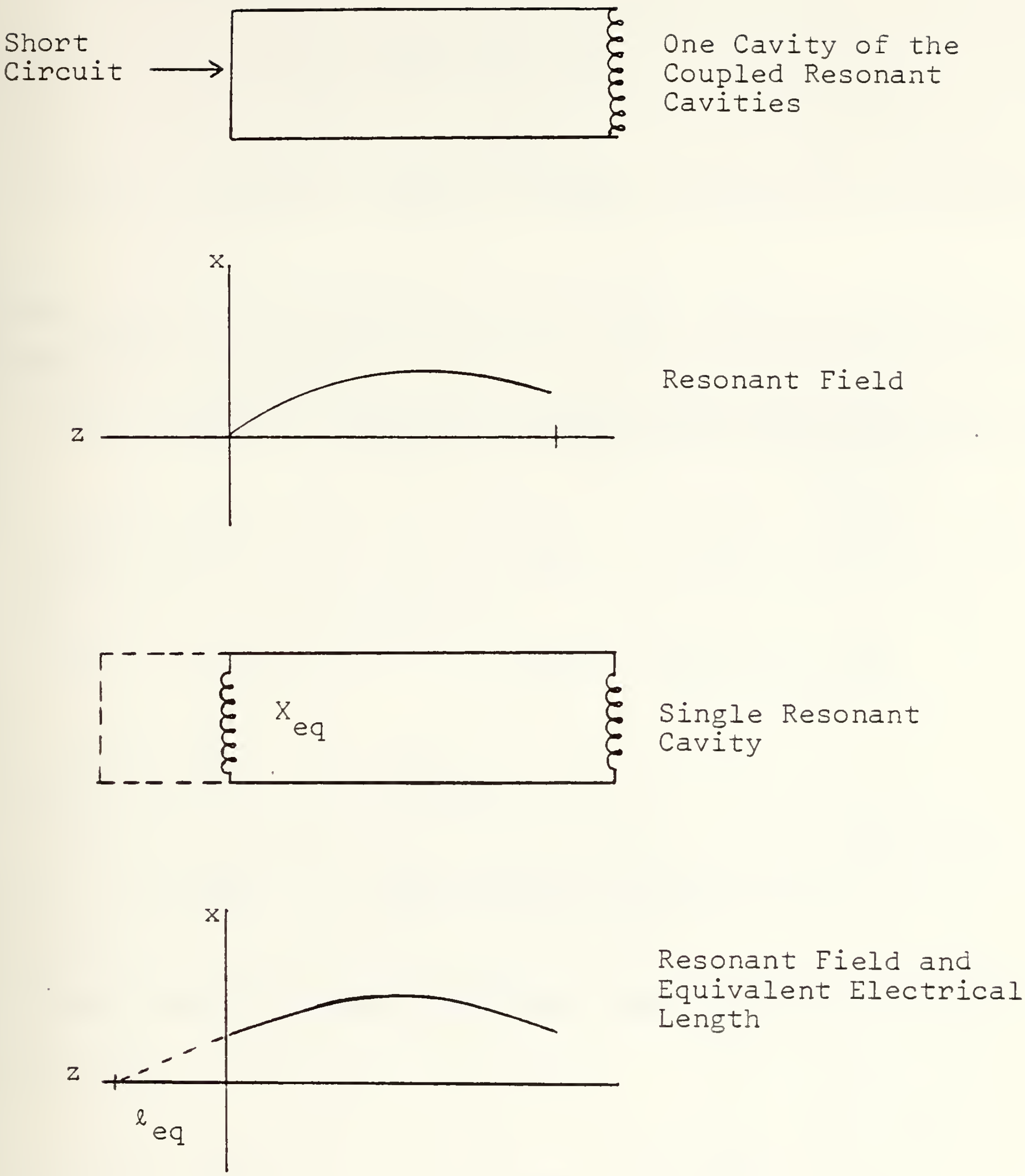


Figure III-3 Equivalent Electrical Length of an Inductive

Therefore, $\ell_o/\lambda' = \ell/\lambda' + \ell_{eq}/\lambda'$

$$x_{eq} = \tan 2\pi\ \ell_{eq}/\lambda'$$

$$\ell_{eq}/\lambda' = \frac{1}{2\pi} \tan^{-1} x_{eq}$$

where x_{eq} = XSC as derived in the single cavity case and is included in Appendix D.

As an example, consider 8 GHz for w/b = 1.0, and t/D = 1.0:

$\ell_o/\lambda' = .487$ $\quad\quad x_{eq} = .098$

$\ell/\lambda' = .469$ $\quad\quad \ell_{eq}/\lambda' = \frac{1}{2\pi} \tan^{-1} .098$

$.487 \cong .469 + .0155$

$\cong .4845$, mean relative error = 0.5%

As a second example, consider 12 GHz for w/b = 1.0 and t/D = 5.0:

$\ell_o/\lambda' = .406$ $\quad\quad x_{eq} = .688$

$\ell/\lambda' = .308$ $\quad\quad \ell_{eq}/\lambda' = \frac{1}{2\pi} \tan^{-1} .688$

$$.406 \cong .308 + .096$$

$$\cong .404, \text{ mean relatives error } = 0.5\%.$$

The mean relative error is defined as:

$$\text{M.R.E.} = \frac{(\ell_o/\lambda') - (\ell/\lambda' + \ell_{eq}/\lambda')}{\sqrt{(\ell_o/\lambda')\ (\ell/\lambda' + \ell_{eq}/\lambda')}}$$

As with the single resonant cavity, two other numerical considerations are singularities in the $E_x^x(\alpha_n)$ and $E_x^z(\xi_k)$ functions and the limits of summation over α_n and ξ_k. Both are derived and resolved in Appendix C. The documented program is found in Appendix H.

The conclusion of this phase of the Fin-Line inductive strip analysis is that numerically the results are correct with regard to convergence and the three checks. Also, the choice of the sine and cosine basis functions was proper and viable. The final phase is to derive the scattering parameters for the inductive strip as a function of both odd and even mode resonant lengths and compare numerical and experimental results.

IV. INDUCTIVE STRIP SCATTERING PARAMETERS

A. THEORY

The final phase of the analysis of the Fin-Line inductive discontinuity consists of deriving its scattering parameters from basic microwave network circuit theory by treating the strip as a lossless two-port network and then comparing the numerically calculated scattering parameters with these determined through experiment. The complete derivation of the scattering parameters is found in Appendix E. Since the inductive strip is considered lossless, recipricol, and symmetric, the scattering matrix is unitary, $S_{11} = S_{22}$ and $S_{21} = S_{12}$. The values of S_{11} and S_{12} are determined to be

$$S_{11} = -\exp(j2\pi[\frac{\ell e}{\lambda'} + \frac{\ell o}{\lambda'}]) \cos 2\pi \, (\frac{\ell e}{\lambda'} - \frac{\ell o}{\lambda'})$$

$$S_{12} = j\exp(j2\pi[\frac{\ell e}{\lambda'} + \frac{\ell o}{\lambda'}]) \sin 2\pi \, (\frac{\ell e}{\lambda'} - \frac{\ell o}{\lambda'})$$

and the equivalent circuit reactances for the inductive strip are determined to be

$$x_s = \frac{1}{2} [\tan 2\pi(\frac{1}{2} - \frac{\ell e}{\lambda'}) + \tan 2\pi(\frac{1}{2} - \frac{\ell o}{\lambda'})]$$

$$M = \frac{1}{2} [\tan 2\pi(\frac{1}{2} - \frac{\ell e}{\lambda'}) - \tan 2\pi(\frac{1}{2} - \frac{\ell o}{\lambda'})]. \quad \text{[Ref. 14]}$$

B. NUMERICAL ANALYSIS AND RESULTS

Using the odd and even mode resonant lengths numerically determined, the corresponding S_{11} and S_{12} parameters are calculated and checked for convergence. Figure IV-1 shows that convergence is achieved with an order of "10" for the $|S_{11}|$ for t/D = 1.0, w/b = 1.0, ε_{r_2} = 1.0, and for 8, 10, and 12 GHz, as an example. Similar convergences were obtained for various other geometric parameters and frequencies for S_{11} θ_{11}, S_{12}, and θ_{12}.

The numerical results obtained herein for S_{11}, θ_{11}, S_{12}, and θ_{12} are compared with the experimental measurements by Miller [Ref. 15] and graphically depicted in Appendix F. Referring to Appendix F, the solid lines with circles are numerical data and the dashed lines with squares are the experimental measurements by Miller. Defining the mean relative error (MRE) between the experimental (EXP) and numerical (NUM) data as:

$$\text{M.Rel.Err.} = \left[\frac{(\text{NUM-EXP})^2}{(\text{NUM})(\text{EXP})}\right]^{1/2} \times 100\%,$$

the mean relative errors for the S_{11} parameters range from 0% up to 8.5% for the phase term θ_{11} for a septum thickness of 0.2 inch at 8 GHz. Errors for the S_{21} parameter are very large in many instances and very erratic.

As an example, for a septum thickness of 0.2 inch, w/b = 1.0 at 8 GHz the mean relative error for the $|S_{21}|$ is

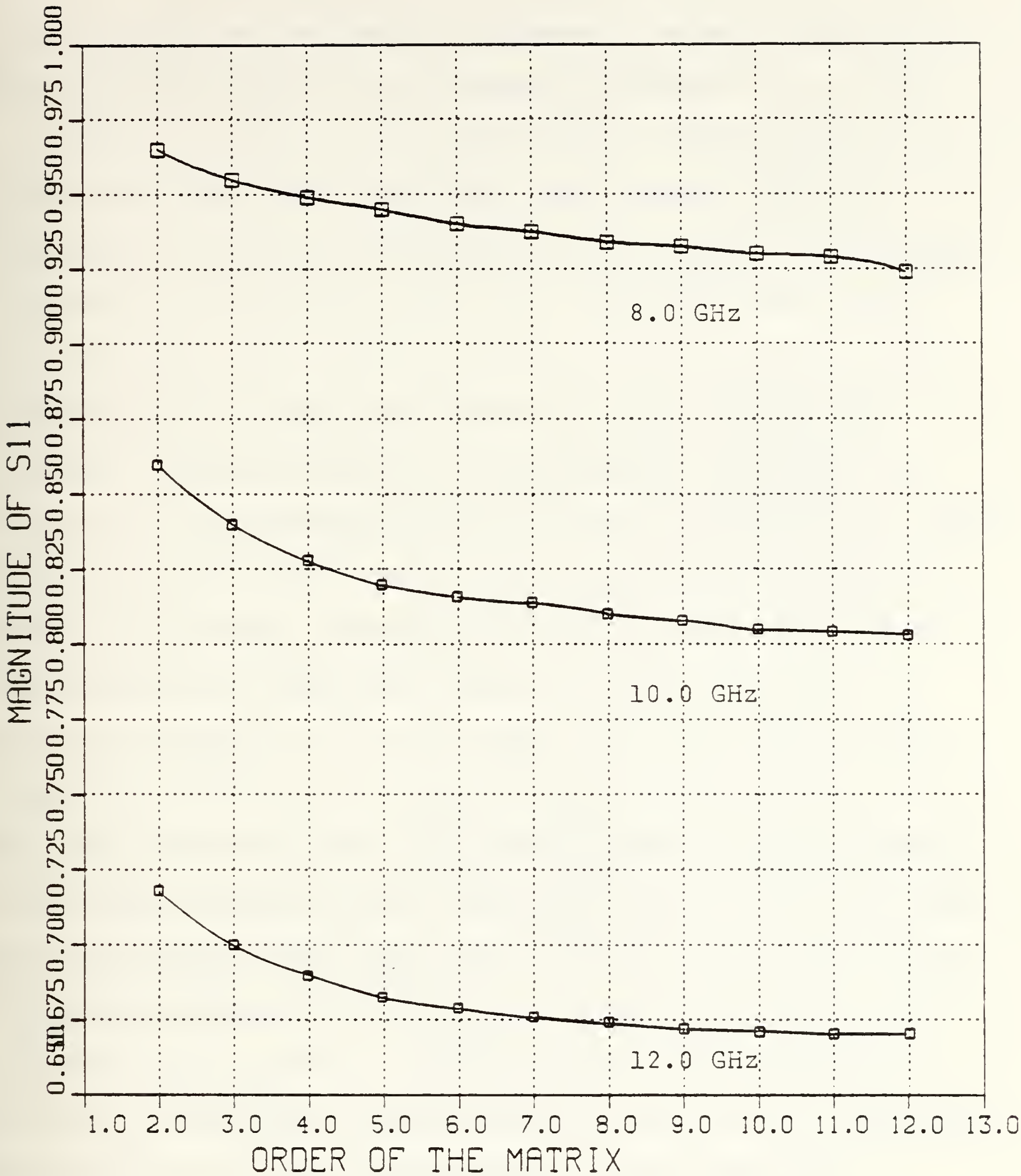


Figure IV-1 $|S_{11}|$ Convergence Test for the Inductive Strip for w/b=1.0, t/D=1.0, and f=8.0, 10.0, and 12.0 GHz

51.4% and for the phase term θ_{21} the error is 86.66%. In view of these two extremes in agreement between experimental and numerical results, some general statements can be made.

For $|S_{11}|$ and θ_{11}, the mean relative error is generally less than 5% and deviations from this general rule are restricted to the lower frequencies near 8 GHz. For the S_{21} parameter, the error for the $|S_{21}|$ follow the same general rule of being less than 5% with the exception of the septum thickness of 0.2 inch which appears to be a measurement anomaly. However, such is not the case for the phase θ_{21} term of the S_{21} parameter where the mean relative error ranges from 0% to 86% with no median value regardless of frequency range. The closest agreement for the S_{21} parameter between experimental and numerical data can be seen to be for the septum thickness of 0.05 inch where the mean relative error ranges from 0% to 7% for the frequency range from 8.0 to 11.5 GHz and 70.7% for 12 GHz. If the inductive strip is indeed a loss less two-port network then the square root of the sum of the squares of the magnitudes of S_{11} and S_{21} should equal 1.0 and the phase terms of S_{11} and S_{21} should differ by 90° degrees, ($\theta_{11}-\theta_{21}=90°$). Considering the close agreement between the numerical and experimental data for the magnitude and phase of S_{11}, it is the author's belief that the experimental results for the magnitude and phase of S_{21} for the inductive strip as presented by Miller in Reference 15 are in error and Miller agrees with this fact in his

conclusions. Since the experimental and numerical data for the S_{11} parameters agree within a 5% error for w/b = 1.0, numerical calculations of the scattering parameter S_{11} were made and plotted for w/b = 0.5, 0.25, 0.2, and 0.1 and are presented in Figures F-25 through F-56 of Appendix F. Values for the S_{21} parameters can be obtained from these curves by noting that:

$$S_{21} = \sqrt{1 - |S_{11}|^2}$$

$$\theta_{21} = \theta_{11} - 90°$$

These curves should prove useful in design and comparison with experimental measurements for w/b < 1.0. The numerical results for w/b < 1.0 presented were obtained using a matrix of order 8, which corresponds to the use of 8 basis functions to represent the electric field spatial distribution, and restricting the maximum limits of summation for the inner product terms which are the elements of the matrix for which a singularity is sought. This was done in the interest of computation time and may represent an error as high as 2% from that which could be obtained from an order of 10 or higher and unrestricted limits of summation over α_n and ξ_k. The numerical results appear physically sound. Comparison of the $|S_{11}|$ reflection coefficient for a specified strip

width t/D for w/b = 1.0 with w/b = 0.5 and 0.25 shows that the $|S_{11}|$ increases in magnitude with decreasing gap width w/b. This is physically reasonable in that as the gap becomes smaller the strip width relative to the gap appears electrically wider and the fields in the two cavities become more uncoupled. Comparison of the $|S_{11}|$ for w/b = 0.25 with 0.2 and 0.1 reveals that the magnitude drops for w/b = 0.2 and then begins to rise again for w/b = 0.1. This may be explained by noting that as the gap becomes smaller the electric field strength within the gap increases and is essentially constant and the multiple reflections of the decaying exponential tails between the two cavities on the strip may increase in strength, correspondingly. Hence, as coupling increases the $|S_{11}|$ decreases. However, experimental measurements must be made in order to determine the "correctness" of the numerical results presented here.

V. CONCLUSIONS AND RECOMMENDATIONS

The spectral domain technique used in conjunction with Galerkin's method and the choice of an expanded set of cosine and sine basis functions to represent the spatial distribution has been demonstrated to be proper, viable, and produce accurate results in the analysis of the fin-line inductive strip. For the single resonant cavity, agreement between previously published numerical and experimental results and the numerical results obtained herein is within 2%. For the coupled cavities using the odd and even mode resonant lengths to determine S_{11} for the inductive strips of various widths the agreement at several checkpoints is within 5%, in general.

Numerical computation time, expecially for $w/b < 1.0$ can be quite large if high accuracy is sought. Numerical accuracy is a function of the number of basis functions used (matrix order), limits of summation over α_n and ξ_k, and the residue decision to determine the end of a search iteration. These three factors, along with search interval directly impact on computation time. As the order of the matrix goes up, so must the accuracy of the elements of matrix which can only be obtained by increasing the limits of summation for the inner product terms. This is due to the fact that as the determinant of the matrix approaches its singularity, which is what is sought, it becomes "ill-conditioned". If the

elements of the matrix contain a small error then an incorrect value of the determinant will result. As the matrix becomes very ill-conditioned, numerically, exponential underflow or overflow will exceed the capabilities of the available IBM 3033 used. Therefore, a trade must be made between accuracy and computation time.

Four recommendations are offered in conclusion. First, due to the computation time required for highly accurate numerical results, it is recommended that the programs contained herein be transferred to a personal minicomputer. Second, experimental measurements should be made for $w/b<1.0$ and compared with the numerical results presented herein. Third, the use of other basis functions should be investigated in depth. Two basis functions are recommended. The close numerical agreement between the basis functions used in this thesis and the cosine with the exponential end correction used by Knorr in Reference 7 indicate that using a single cosine function with a sum of exponentials representing the decaying tails of higher order modes may prove to be very accurate. The second basis set recommended is formed from the Chebyshev polynomials of the first kind, $T_n(x)$, presented here as

$$F_n(x) = xT_n(x) - T_{n+1}(x).$$

This is a modified Chebyshev polynomial of the second kind, defined over the interval of -1 to +1 and is zero at these two points regardless of order. This function has the advantage of being even or odd depending on the value of the index "n". The fourth and final recommendation is that a faster search routine be sought. The bisectional search used herein is the most accurate and reliable but has the burden of being the slowest. The Newton-Gregory search method is offered for consideration.

APPENDIX A

THEORETICAL ANALYSIS OF THE SINGLE RESONANT CAVITY

Derivation of the Single Resonant Cavity Problem

Begin with
$$G_{11}E_x + G_{12}E_z = J_x$$

$$G_{21}E_x + G_{22}E_z = J_z$$

where all terms are in the frequency domain and the G_{11}, G_{12}, G_{21}, and G_{22} are the dyadic Green's functions as obtained from Knorr [7].

Mult by E_x and E_z and taking the inner product:

$$\langle G_{11}\, E_x, E_x\rangle + \langle G_{12}\, E_z, E_x\rangle = \langle J_x, E_x\rangle$$

$$\langle G_{21}\, E_x, E_z\rangle + \langle G_{22}\, E_z, E_z\rangle = \langle J_z, E_z\rangle$$

Next assuming $E_z \approx 0$

$$\langle G_{11}\, E_x, E_x\rangle = \langle J_x, E_x\rangle$$

Here the inner product is defined as

$$\langle a, b\rangle = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} a(\alpha_n, \xi_k)\, b^*(\alpha_n, \xi_k)$$

Substituting $E_x = \Sigma B'_n E_{x_n}$ into the inner product equation and multiplying by E_{x_m} :

$$\langle E_{x_m}, G_{11} \Sigma B'_n E_{x_n} \rangle = \langle J_x, E_{x_m} \rangle = 0$$

which reduces to

$$\begin{bmatrix} & & \\ & \langle E_{x_i}, G_{11} E_{x_j} \rangle & \\ & & \end{bmatrix} \begin{bmatrix} B'_1 \\ B'_2 \\ \cdot \\ \cdot \\ \cdot \\ B'_M \end{bmatrix} = 0$$

i and j
go from 1 to M

Clearly, if all B''s are zero a trivial solution results $\therefore$ solve the characteristic equation

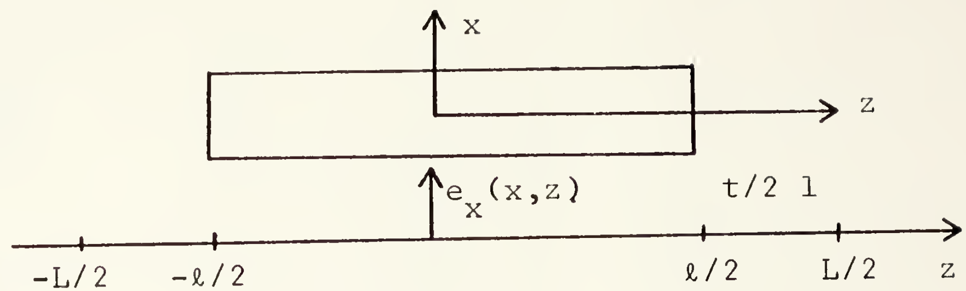
$$\det[\langle\ \rangle] = 0$$

Assume: $E_x(\alpha_n, \xi_k) = E_x^x(\alpha_n) E_x^z(\xi_k) = \Sigma B'_m E_{x_m}(\alpha_n, \xi_k)$

Let $e_x(x,z) = A \sum_{m=1}^{M} B_m \cos \frac{(2m-1)\pi z}{\ell}$

for $|x| \leq w/2$

$|z| \leq \ell/2$



$L = \ell + t$

Define

$$E_x^x(\alpha_n) = F[e_x^x(x,z)] = \int_{-w/2}^{w/2} A\, e^{j\alpha_n x}\, dx = AW \frac{\sin\frac{1}{2}(\alpha_n D)(w/D)}{\frac{1}{2}(\alpha_n D)(w/D)}$$

Define

$$\Sigma B_m E_{x_m}^z(\xi_k) = F]e_x^z(x,z)] = \int_{-\ell/2}^{\ell/2} \Sigma B_m \cos \frac{(2m-1)\pi z}{\ell}\, e^{j\xi_k z}\, dz$$

$$\sum_{m=1}^{M} B_m (-1)^m \frac{(2m-1)}{2} \ell\pi \frac{\cos(\xi_k D)(\ell/2D)}{(\xi_k D)^2 (\ell/2D)^2 - [\frac{(2m-1)\pi}{2}]^2}$$

Determining the actual value of B'_m to be discarded:

$$E_{x_m} = \Sigma\ AW\left[\frac{\sin\frac{1}{2}(\alpha_n w)}{\frac{1}{2}(\alpha_n w)}\right]B_m(-1)^m \frac{(2m-1)}{2}\ \ell\pi$$

$$X \quad \frac{\cos\frac{1}{2}(\xi_k \ell)}{(\xi_u \ell/2)^2 - [\frac{(2m-1)\pi}{2}]^2}$$

Therefore, $B'_m = \frac{1}{2} AWB_m \ell\pi$.

And the inner product for the ijth term becomes

$$\sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} G_{11}\left[\frac{\sin\frac{1}{2}(\alpha_n w)}{\frac{1}{2}(\alpha_n w)}\right]^2 (-1)^{i+j}(2i-1)(2j-1) \frac{\cos\frac{1}{2}(\xi_k \ell)}{(\xi_k \ell/2)^2 - [\frac{(2i-1)\pi}{2}]^2}$$

$$X \quad \frac{\cos\frac{1}{2}(\xi_k \ell)}{(\xi_k \ell/2)^2 - [\frac{(2j-1)\pi}{2}]^2}$$

The inner product in expanded form can be written as:

$$\sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} f(\alpha_n,\xi_k) = f(\alpha_o,\xi_o) + 2 \sum_{n=1}^{n_{max}} f(\alpha_n,\xi_o) + 2 \sum_{k=1}^{k_{max}} f(\alpha_o,\xi_k)$$

$$+ 4 \sum_{n=1}^{n_{max}} \sum_{k=1}^{k_{max}} F(\alpha_n,\xi_k)$$

which can be reduced to:

$$2 \sum_{k=1}^{k_{max}} f(\alpha_n,\xi_k) + 4 \sum_{n=1}^{n_{max}} \sum_{k=1}^{k_{max}} f(\alpha_n,\xi_k), \text{ if f is even.}$$

N_{max} is determined by noting $|E_x^x(\alpha_n)| \sim \frac{1}{\frac{1}{2}\alpha_n w}$

Letting $\frac{1}{2}\alpha_n w = \frac{xconst}{2}$ where $\alpha_n = \frac{n2\pi}{b}$

yields $N_{max} = \frac{xconst}{2\pi} b/w$

And K_{max} is determined by noting that

$$|E_x^z(\xi_k)| \sim \frac{1}{\theta^2 - [\frac{(2m-1)\pi}{2}]^2} \qquad \text{where} \quad \theta = \xi_k\, \ell/2$$

Letting $\theta^2 - \frac{(2m-1)^2\pi^2}{4} = \text{zconst}\ \pi^2$

and $\xi_k = (2k-1)\pi/L$ where $L = \ell + t$

yields $K_{max} = \frac{1}{2} + [\text{zconst} + \frac{(2m-1)^2}{4}]^{1/2} (1 + t/\ell)$

The singularities of the E_x^x and E_x^z functions are determined via L'Hospital's Rule:

$$\lim_{\phi \to 0} \frac{\sin\phi}{\phi} = 1.0, \quad \text{where } \phi = \frac{1}{2}\alpha_n w$$

$$\lim_{\theta \to \frac{(2m-1)\pi}{2}} \frac{\cos\theta}{\theta^2 - [\frac{(2m-1)\pi}{2}]^2} = \frac{(-1)^m}{(2m-1)\pi}, \quad \text{where } \theta = \xi_k\, \ell/2$$

The G_{11} dyadic Green's function obtained from Knorr, notes for Reference 7, is defined as

$$G_{11} = g_{11}^{a} + g_{11}^{b} + g_{11}^{c} + g_{11}^{d} + g_{11}^{e}$$

$$g_{11}^{a} = -j\left(\frac{(k_{c_1}D)^2}{(\omega\mu D)(\gamma_1 D)\tanh[(\gamma_1 D)(h_1/D)]}\right)$$

$$g_{11}^{b} = -j\left(\frac{(k_{c_2}D)\, d_{12}}{d_{11}d_{22}-d_{21}d_{12}}\right)\left(\frac{(\alpha_n D)(\beta D)(k_{c_2}D)^2\,(\gamma_2 D)\,\tanh(\gamma_2 D)}{(\omega\mu D)(\gamma_2 D)^2(k_{c_3}D)^2}\right)$$

$$g_{11}^{c} = j\left(\frac{d_{12}(k_{c_2}D)^2}{d_{11}d_{22}-d_{21}d_{12}}\right)\left(\frac{(\alpha_n D)(\beta D)(\gamma_2 D)\,\tanh(\gamma_2 D)}{(\omega\mu D)(\gamma_2 D)^2}\right)$$

$$g_{11}^{d} = -j\left(\frac{d_{11}(k_{c_2}D)^2}{d_{11}d_{22}-d_{21}d_{12}}\right)\left(\frac{(k_{c_2}D)^2(\gamma_3 D)\,\tanh[(\gamma_3 D)(h_2/D)]}{(k_{c_3}D)^2(\gamma_2 D)^2}\right)$$

$$g_{11}^{e} = -j\left(\frac{(k_{c_2}D)^2\, d_{11}}{(d_{11}d_{22}-d_{21}d_{12})(\gamma_2 D)\tanh(\gamma_2 D)}\right)$$

where

$$d_{11} = -(k_{c_2}D)^2\left[1 + \frac{(\omega\varepsilon_3 D)(\gamma_3 D)(k_{c_2}D)^2 \tanh(\gamma_2 D)}{(\omega\varepsilon_2 D)(\gamma_2 D)(k_{c_2}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}\right]$$

$$d_{12} = (k_{c_2}D)^2\left[\frac{(\alpha_n D)(\beta D)}{(\omega\varepsilon_2 D)(\gamma_2 D)^2}\right]\left[\frac{(k_{c_2}D)^2}{(k_{c_3}D)^2} - 1\right]$$

$$d_{21} = -(\alpha_n D)(\beta D)\left[\frac{(k_{c_2}D)^2}{(k_{c_1}D)^2} + \frac{(\omega\varepsilon_3 D)(\gamma_3 D)(k_{c_2}D)^2 \tanh(\gamma_2 D)}{(\omega\varepsilon_2 D)(\gamma_2 D)(k_{c_3}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}\right]$$

$$d_{22} = (w\mu D)\left[1 + \frac{(\gamma_3 D)(k_{c_2}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2 D)(k_{c_3}D)^2 \tanh(\gamma_2 D)} + \frac{(\alpha_n D)(\beta D)}{(\omega\mu D)(\gamma_2 D)}\right.$$

$$\left. X\left(\frac{(\alpha_n D)(\beta D)(k_{c_2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2 D)(k_{c_3}D)^2} - \frac{(\alpha_n D)(\beta D)}{(\omega\varepsilon_2 D)(\gamma_2 D)^2}\right)\right]$$

$$(k_{c_1}D)^2 = (2\pi)^2\,[\varepsilon_{r_1} - (\lambda/\lambda')^2](D/\lambda)^2$$

$$(k_{c_2}D)^2 = (2\pi)^2\,[\varepsilon_{r_2} - (\lambda/\lambda')^2](D/\lambda)^2$$

$$(k_{c_3}D)^2 = (2\pi)^2\,[\varepsilon_{r_3} - (\lambda/\lambda')^2](D/\lambda)^2$$

$$\omega\mu D = 240\ {}^{2}(D/\lambda)$$

$$\omega\varepsilon_1 D = {}_{r_1}/60(D/\lambda)$$

$$\omega\varepsilon_2 D = {}_{r_2}/60(D/\lambda)$$

$$\omega\varepsilon_3 D = {}_{r_3}/60(D/\lambda)$$

Assume $\mu_1 = \mu_2 = \mu_3 = \mu_0$

$$(\gamma_1 D)^2 = (\alpha_n D)^2 + (2\pi)^2[(\lambda/\lambda') - \varepsilon_{r_1}](D/\lambda)^2$$

$$(\gamma_2 D)^2 = (\alpha_n D)^2 + (2\pi)^2[(\lambda/\lambda') - \varepsilon_{r_2}](D/\lambda)^2$$

$$(\gamma_3 D)^2 = (\alpha_n D)^2 + (2\pi)^2[(\lambda/\lambda') - \varepsilon_{r_3}](D/\lambda)^2$$

If $(\gamma_i D) < 0$

then define

$$(\gamma_1'' D)^2 = -(\gamma_1 D)^2$$

$$(\gamma_2'' D)^2 = -(\gamma_2 D)^2$$

$$(\gamma_3'' D)^2 = -(\gamma_3 D)^2$$

$$\beta D = -\xi_k D$$

$\alpha_n D = 2n\pi(D/b)$, see Reference 7

$\xi_k D = (2k-1)\pi/(L/D)$, see Reference 7

APPENDIX B

NUMERICAL RESULTS FOR THE SINGLE RESONANT CAVITY

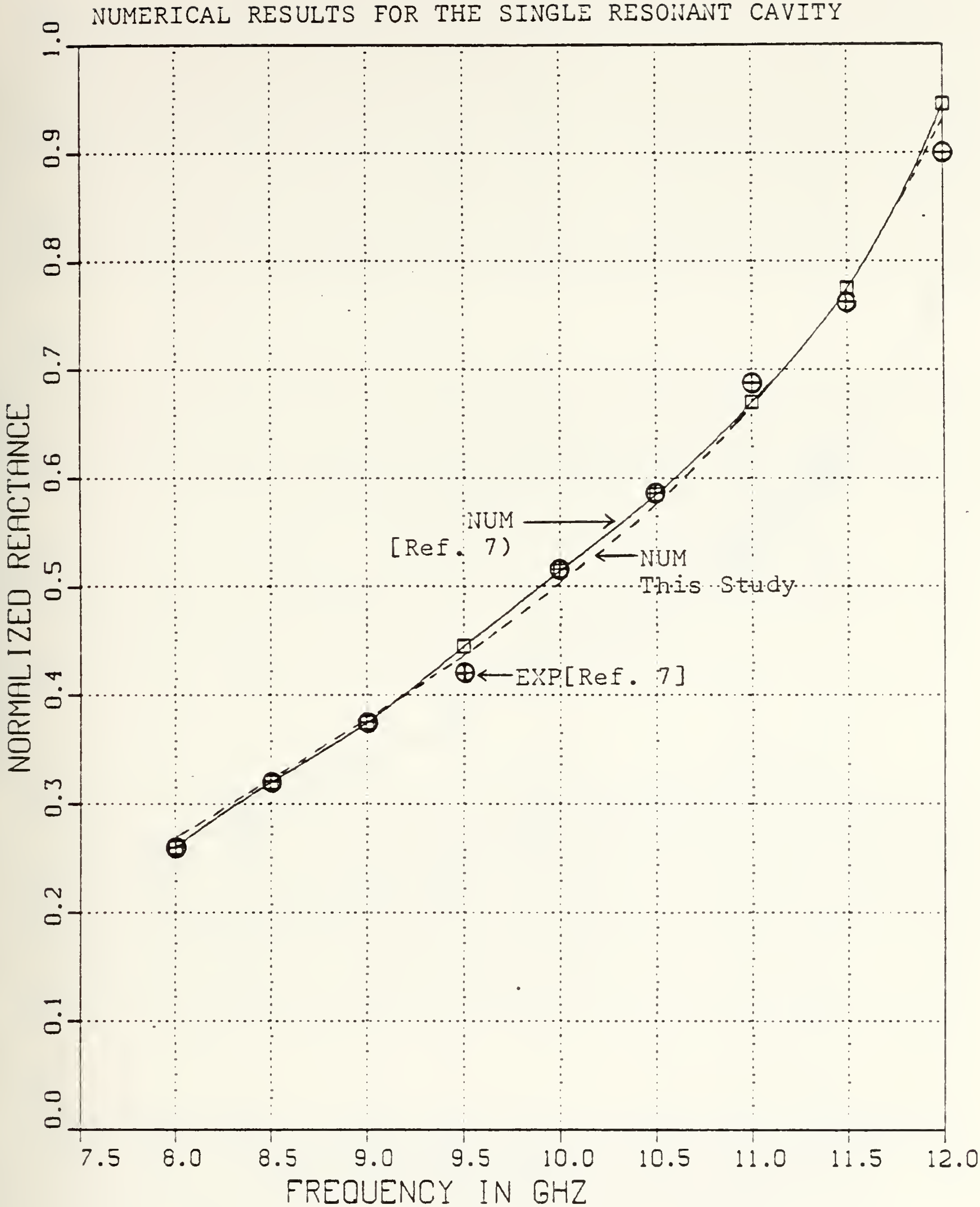


Figure B-1 Normalized Septum Reactance v.s. Frequency for ε_{r_2} =1.0, and w/b=1.0 as the Inductive Strip Width $T \to \infty$

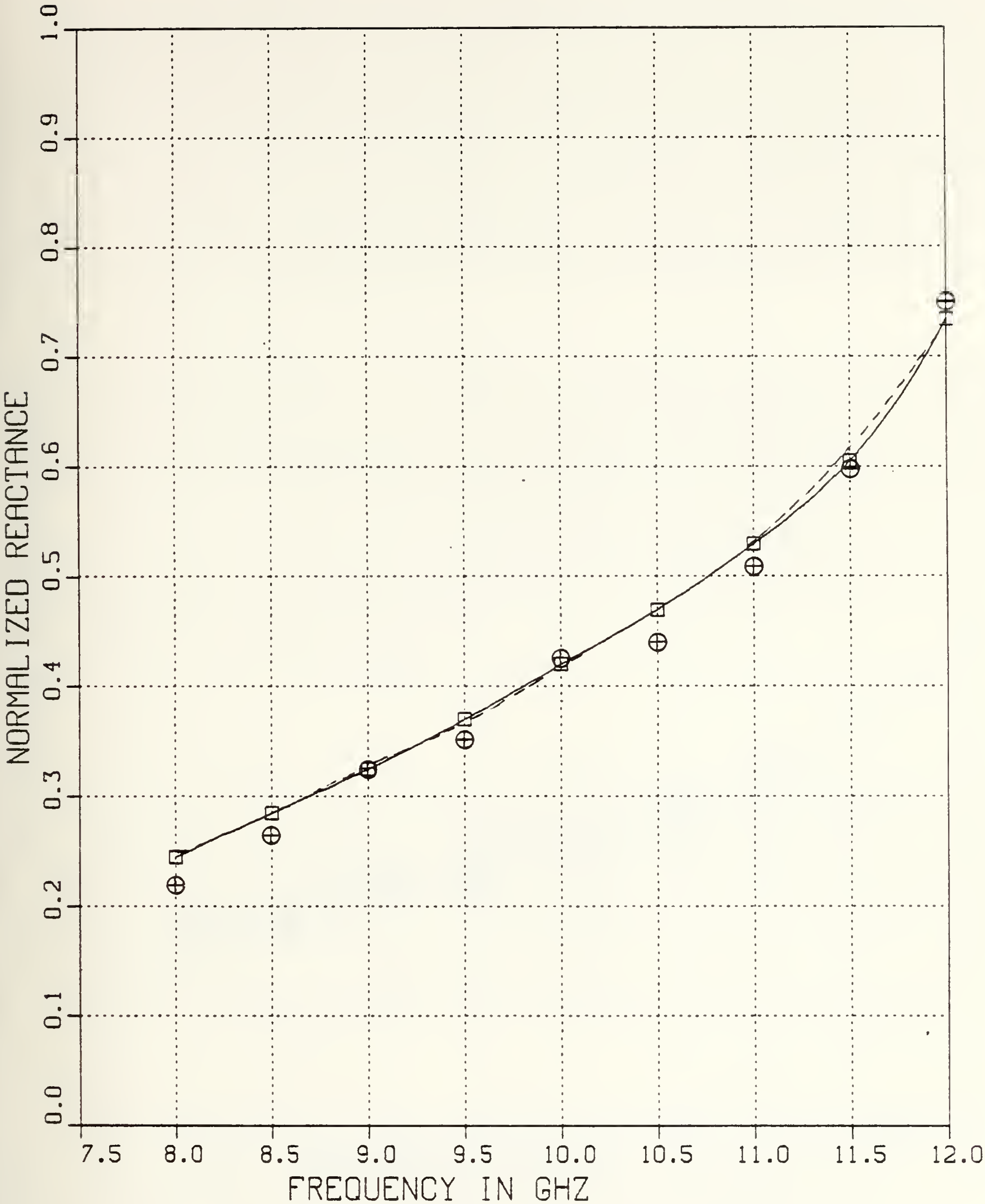


Figure B-2 Normalized Septum Reactance v.s. Frequency for ε_{r_2}=1.0 and w/b=0.5 as the Inductive Strip Width $T\to\infty$

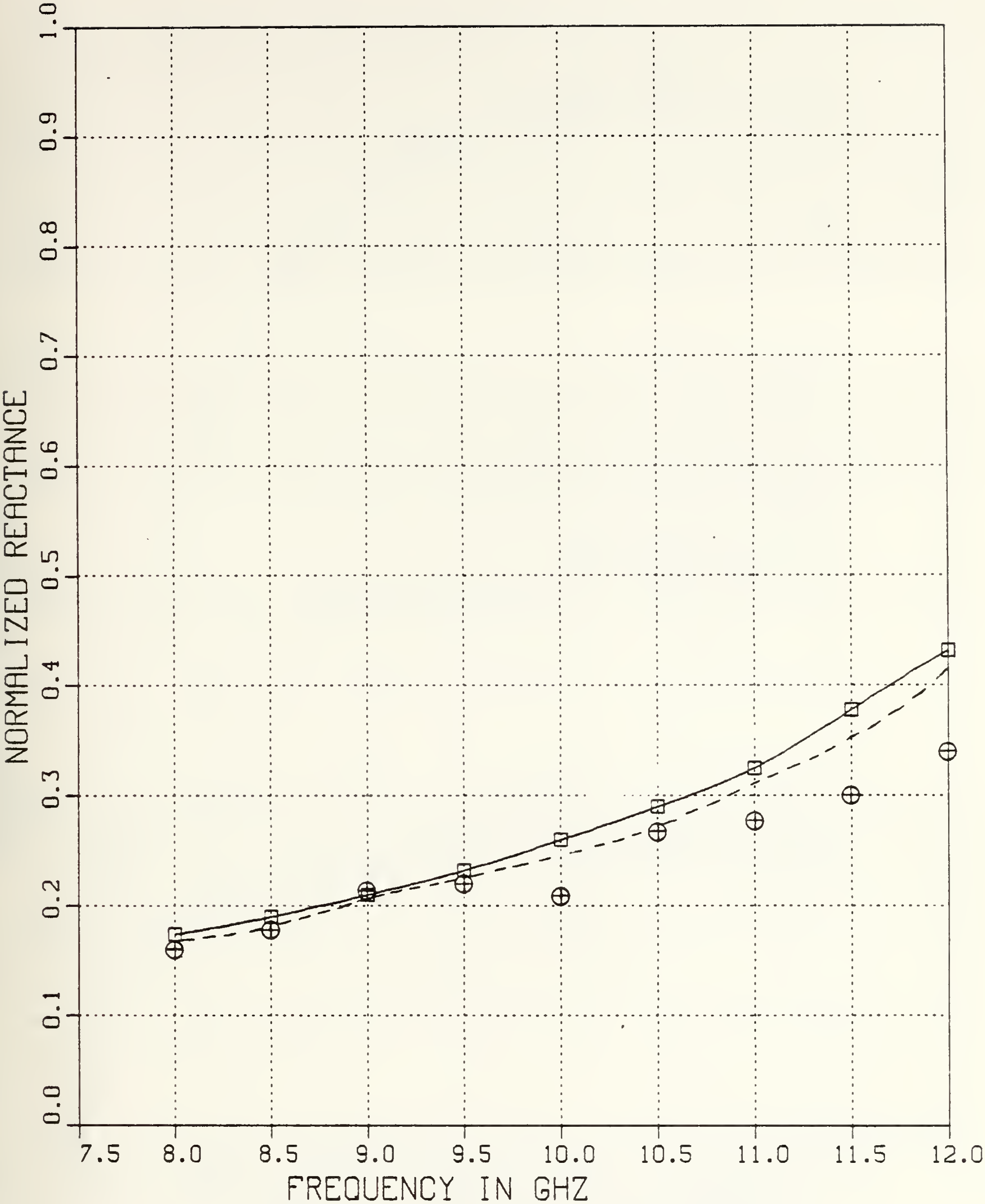


Figure B-3 Normalized Septum Reactance v.s. Frequency for $\varepsilon_{r_2}=1.0$ and w/b=0.1 as the Inductive Strip Width $T\to\infty$

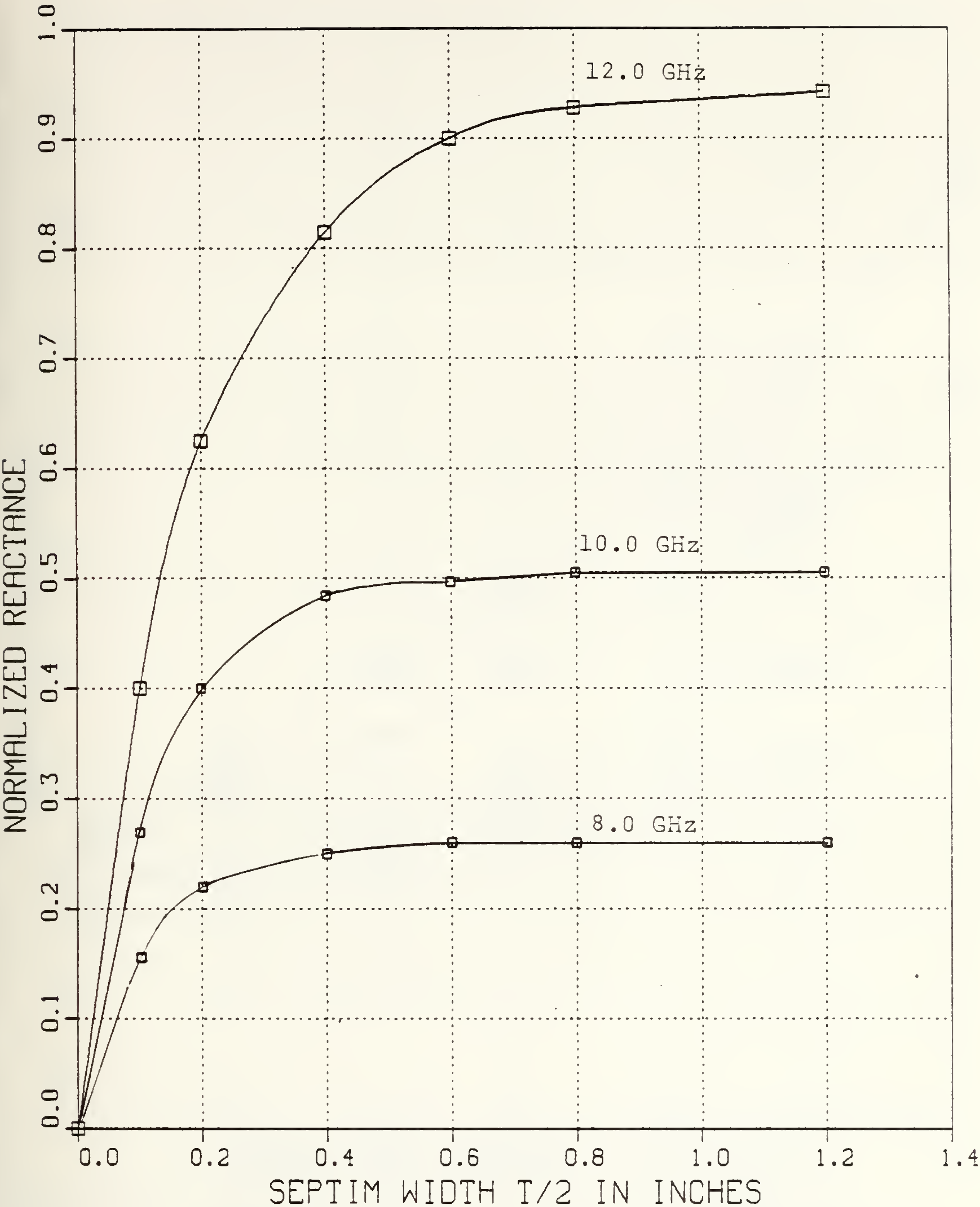


Figure B-4 Normalized Septum Reactance v.s. Septum Length T/2 in Inches for w/b=1.0 and ε_{r_2}=1.0 for 8, 10, and 12 GHz

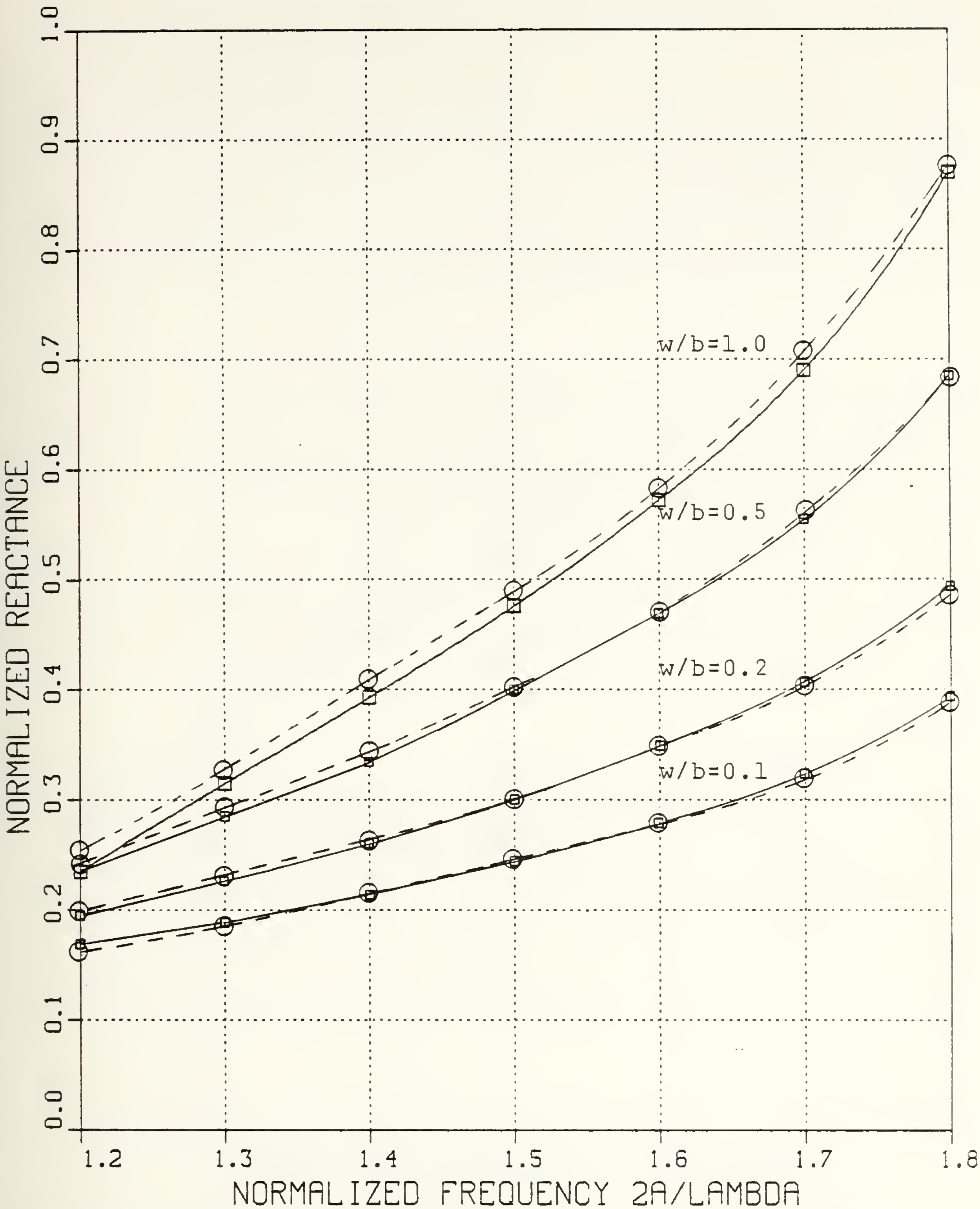


Figure B-5 Normalized Septum Reactance v.s. Normalized Frequency for $\varepsilon_{r_2}=2.2$, b/a=0.5, $h_1/a=0.5$, D/a=0, as $T\to\infty$

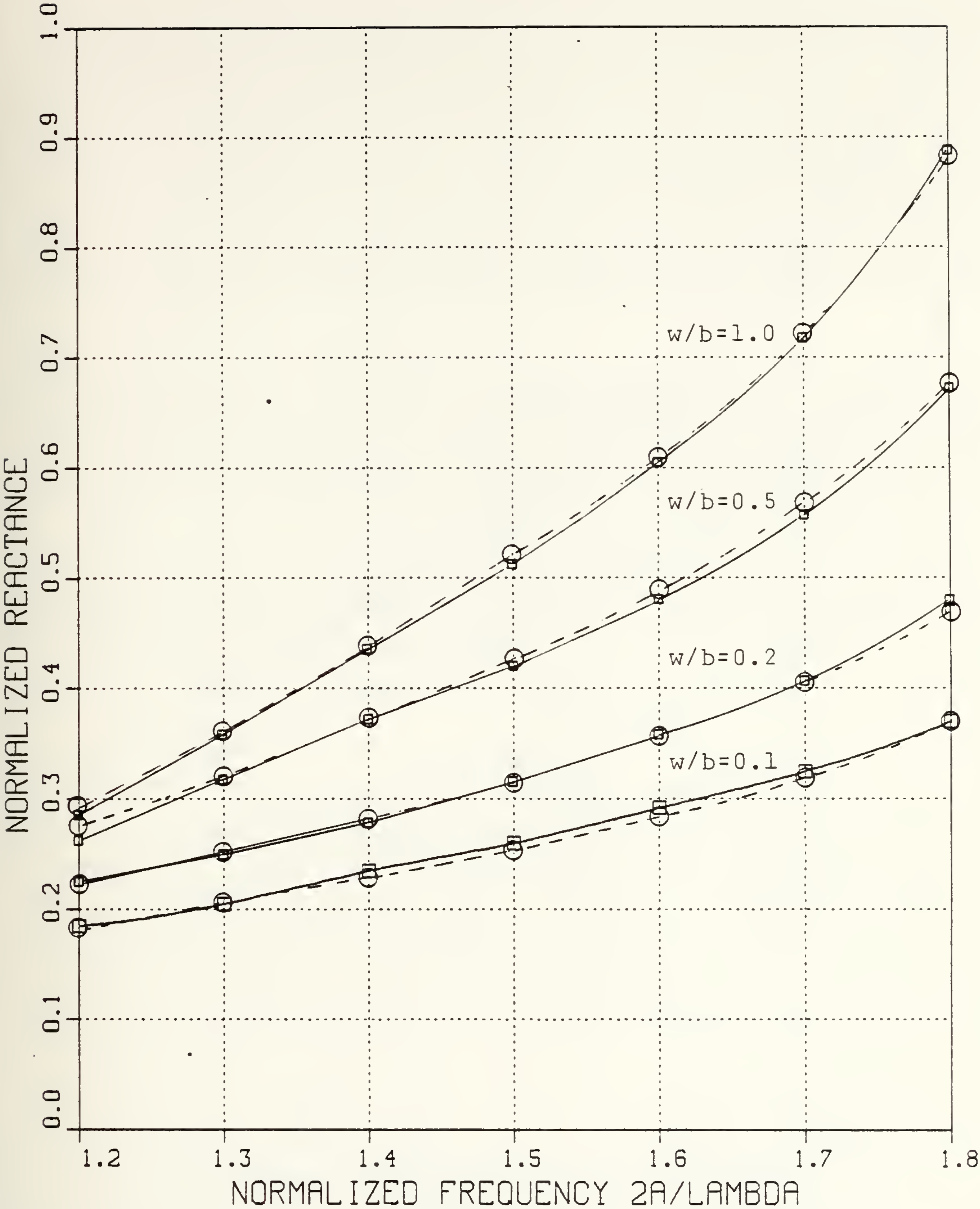


Figure B-6 Normalized Septum Reactance v.s. Normalized Frequency for $\varepsilon_r = 2.2$, $b/a=0.5$, $h_1/a=0.5$, $D/a=0.05$, as $T \to \infty 2$

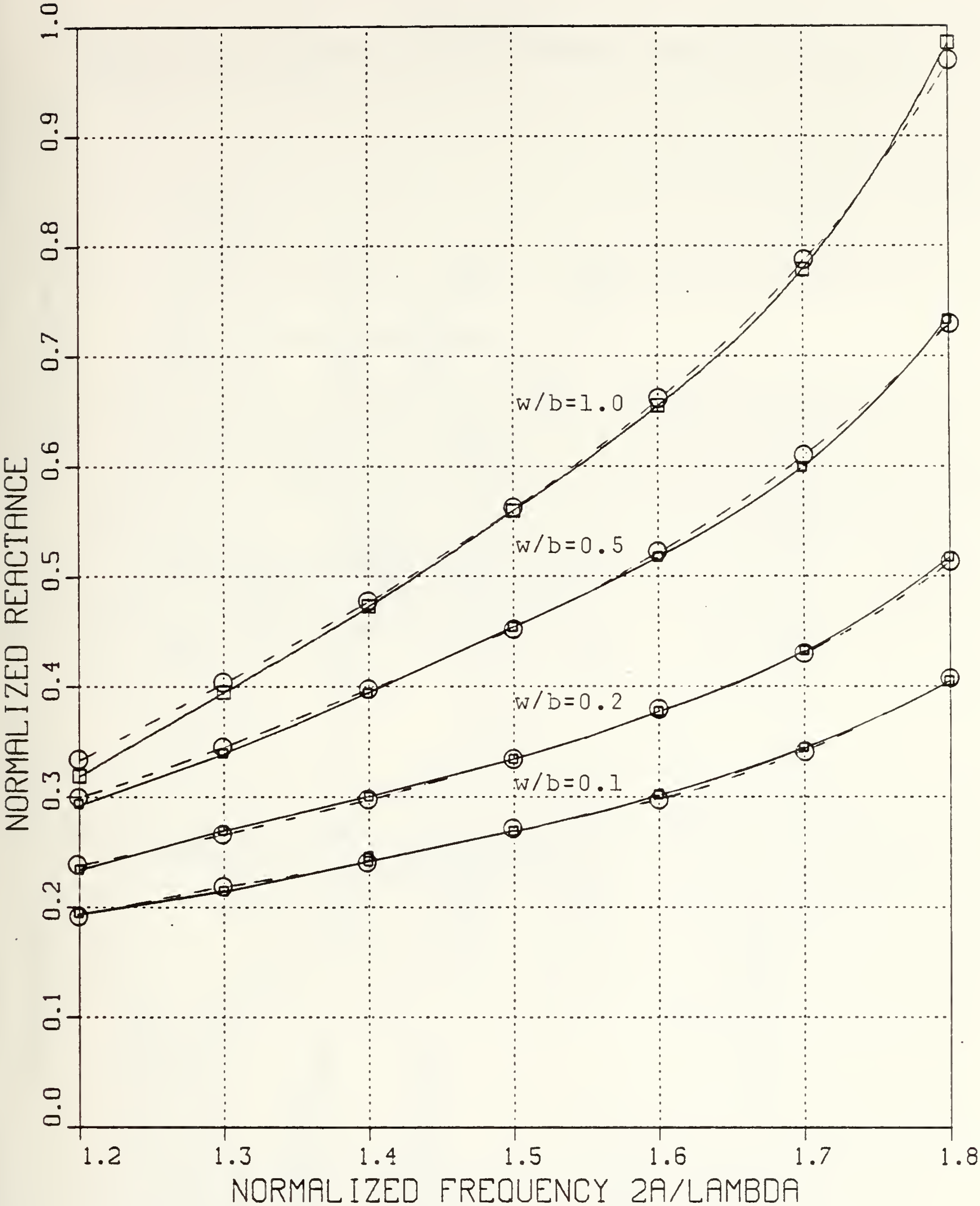


Figure B-7 Normalized Septum Reactance v.s. Normalized Frequency for $\varepsilon_{r_2}=2.2$, b/a=0.5, $h_1/a=0.5$, D/a=0.1, as $T\rightarrow\infty$

APPENDIX C

THEORETICAL ANALYSIS OF THE COUPLED RESONANT CAVITIES

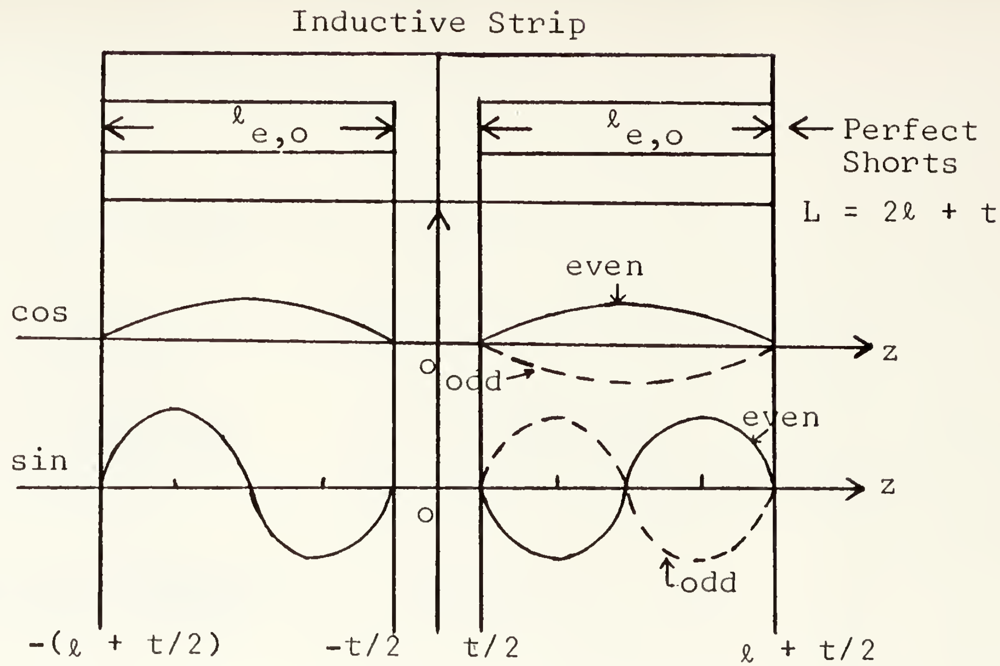
Derivation of the Coupled Resonant Cavities Problem

For the two coupled cavities, two basis functions are used for the odd and even modes where

$\xi_k = (2k-1)\pi/L$
  for even mode

$\xi_k = k2\pi/L$
  for odd mode

as specified in Reference 7.



Here

$$e(x,z) = A \sum_{q=1}^{Q} B_q F_q(z) \quad , \quad |x| \le w/x$$
$$, \quad t/2 \le |z| \le \ell + t/2$$

where

$$
Fq(z) = \begin{cases}
\cos \frac{q\pi}{\ell} \left[z + \left(\frac{\ell + t}{} \right) \right] & , \; z < 0 \\[2ex]
\pm \cos \frac{q\pi}{\ell} \left[z - \left(\frac{\ell + t}{2} \right) \right] & , \; z > 0 \\[1ex]
\quad \text{for } q \text{ odd} & \\
\quad + \text{ even mode} & - \text{ odd mode} \\[2ex]
- \sin \frac{q\pi}{\ell} \left[z + \left(\frac{\ell + t}{2} \right) \right] & , \; z < 0 \\[2ex]
\pm \sin \frac{q\pi}{\ell} \left[z - \left(\frac{\ell + t}{2} \right) \right] & , \; z > 0
\end{cases}
$$

for q even

+ even mode — odd mode

The Fourier transform of $e_x(x)$, (x dependence), gives the same $\mathcal{E}_x^x(\alpha_n)$ as in the single cavity case.

Define $\mathcal{E}_{x_q}^{z_c}(\xi_k)$ as the Fourier Transform of F for the cosine basis function

$\mathcal{E}_{x_q}^{z_s}(\xi_k)$ as the Fourier Transform of F for the sine basis function

Noting that $f(t+T) \leftrightarrow F(f)e^{j2\pi fT}$

$$f(t-T) \leftrightarrow F(f)e^{-j2\pi fT}$$

and that the Fourier Transform of the z-component is defined the same as in the single cavity base, for the cosine function:

$$\mathcal{E}_{x_q}^{z_c}(\xi_k) = (-1)^q \frac{q}{2} \ell\pi \frac{\cos(\xi_k \ell/2)}{(\xi_k \ell/2)^2 - (\frac{q\pi}{2})^2} \quad e^{j\xi_k(\frac{\ell+t}{2})}$$

$$\text{for } z < 0$$

$$E_{x_q}^{z_c}(\xi_k) = \pm (-1)^q \frac{q}{2} \ell\pi \frac{\cos(\xi_k \ell/2)}{(\xi_k \ell/2)^2 - (\frac{q\pi}{2})^2} e^{-j\xi_k(\frac{\ell+t}{2})}$$

for $z > 0$

Therefore, for the even mode:

$$E_{x_q}^{z_c} = (-1)^q \frac{q}{2} \ell\pi \frac{\cos(\xi_k \ell/2)}{(\xi_k \ell/2)^2 - (\frac{q\pi}{2})^2} [e^{j\xi_k(\frac{\ell+t}{2})} + e^{-j\xi_k(\frac{\ell+t}{2})}]$$

$$\text{or } [\qquad\qquad] (2) \cos \xi_k(\frac{\ell+t}{2})$$

and for the odd mode

$$E_{x_q}^{z_c} = [\qquad\qquad] (2j) \sin\xi_k(\frac{\ell+t}{2})$$

obtained by summing the two transforms $E_{x_q}^{z_c}$'s for $z<0$ and $z>0$.

and for the sine function:

$$E_{x_q}^{z_s}(\xi_k) = j(-1)^{q+1}(\tfrac{q\pi}{2}\ell)\,\frac{\sin(\xi_k\ell/2)}{(\xi_k\ell/2)^2-(\frac{q}{2}\pi)^2}\,e^{j\xi_k(\frac{\ell+t}{2})}$$

for $z < 0$

$$E_{x_q}^{z_s}(\xi_k) = \pm\, j(-1)^{q}(\tfrac{q}{2}\pi\ell)\,\frac{\sin(\xi_k\ell/2)}{(\xi_k\ell/2)^2-(\frac{q}{2}\pi)^2}\,e^{-j\xi_k(\frac{\ell+t}{2})}$$

for $z > 0$

Therefore, for the even mode:

$$E_{x_q}^{z_s} = (-1)^q(\tfrac{q}{2}\pi\ell)\,\frac{\sin(\xi_k\ell/2)}{(\xi_k\ell/2)^2-(\frac{q}{2}\pi)^2}\,(2)\,\sin\xi_k(\tfrac{\ell+t}{2})$$

and for the
odd mode

$$E_{x_q}^{z_s} = [\qquad\qquad]\,(-2j)\cos\xi_k(\tfrac{\ell+t}{2})$$

obtained by summing the two transforms $E_{x_q}^{z_s}$'s for $z<0$ and $z>0$.

Define $\mathcal{F}[A \sum_{q=1}^{Q} B_q F_q(z)] = \sum_{q=1}^{Q} \tilde{B}_q \mathcal{E}_{x_q}(\alpha_n, \xi_k)$

proceeding as in the single cavity case:

$$\langle \mathcal{E}_{x_i}, G_{11} \sum^{Q} \tilde{B}_j \mathcal{E}_{x_j} \rangle = 0 \quad , \text{ i and j} = 1 \text{ to } Q$$

$$\text{where } \tilde{B} = AB$$

expanding

$$\langle \mathcal{E}^x_x \mathcal{E}^x_x \; G_{11} \mathcal{E}^z_{x_1}, \tilde{B}_1 \mathcal{E}^z_{x_1} \rangle + \ldots\ldots\ldots \langle \mathcal{E}^x_x \mathcal{E}^x_x \; G_{11} \mathcal{E}^z_{x_1}, \tilde{B}_Q \mathcal{E}^z_{x_Q} \rangle = 0$$

$$\vdots \qquad\qquad\qquad\qquad\qquad \vdots$$

$$\langle \mathcal{E}^x_x \mathcal{E}^x_x \; G_{11} \mathcal{E}^z_{x_Q}, \tilde{B}_1 \mathcal{E}^z_{x_1} \rangle + \ldots\ldots\ldots \langle \mathcal{E}^x_x \mathcal{E}^x_x \; G_{11} \mathcal{E}^z_{x_Q}, \tilde{B}_Q \mathcal{E}^z_{x_Q} \rangle = 0$$

substituting in the sine and cosine functions

$$\langle \mathcal{E}^x_x \mathcal{E}^x_x \; G_{11} \mathcal{E}^{z\,c}_{x_1}, \tilde{B}_1 \mathcal{E}^{z\,c}_{x_1} \rangle + \langle \mathcal{E}^{x^2}_x \; G_{11} \mathcal{E}^{z\,c}_{x_1} \tilde{B}_2 \mathcal{E}^{z\,s}_{x_2} \rangle + \langle \mathcal{E}^{x^2}_x \; G_{11} \mathcal{E}^{z\,c}_{x_1} \tilde{B}_3 \mathcal{E}^{z\,c}_{x_3} \rangle$$

$$+ \langle \mathcal{E}^{x^2}_x \; G_{11} \mathcal{E}^{z\,c}_{x_1} \tilde{B}_4 \mathcal{E}^{z\,s}_{x_4} \rangle + \ldots\ldots\ldots = 0$$

$$\vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots$$

$$\langle {E^x_x}^2 \; G_{11} E^{z_{s,c}}_{x_Q}, \tilde{B}_1 E^{z_c}_{x_1} \rangle + \cdots\cdots\cdots \langle {E^x_x}^2 \; G_{11} E^{z_{s,c}}_{x_Q}, \tilde{B}_Q E^{z_{s,c}}_{x_Q} \rangle = 0$$

where $E^{z_{s,c}}_{x_q}$ is the transformed sine function if q is even and the transformed cosine function is q is odd

This expansion reduces to:

$$\begin{bmatrix} \\ \langle E^x_x E^x_x \; G_{11} E^{z_{s,c}}_{x_i}, \; E^{z_{s,c}}_{x_j} \rangle \\ \\ \end{bmatrix} \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \\ \vdots \\ \tilde{B}_Q \end{bmatrix} = 0$$

where i and j run from 1 to Q.

As in the single cavity case, the problem reduces to

$$\det[\langle\ \rangle] = 0$$

Looking at each "ξ_k" inner product term for the even and odd modes, respectively:

For Even Mode:

$$E^{z}_{x_i}{}^{s} E^{z}_{x_j}{}^{c} = (\pi \ell)^2 [(-1)^i (i) \frac{\sin\theta}{\theta^2 - (\frac{i}{2}\pi)^2} \sin\phi \; (-1)^j \; j \frac{\cos\theta}{\theta^2 - (\frac{j\pi}{2})^2} \cos\phi]$$

$$E^{z}_{x_i}{}^{s} E^{z}_{x_j}{}^{s} = (\pi \ell)^2 [(-1)^i (i) \frac{\sin\theta}{\theta^2 - (\frac{i}{2}\pi)^2} \sin\phi \; (-1)^j (j) \frac{\sin\theta}{\theta^2 - (\frac{j\pi}{2})^2} \sin\phi]$$

$$E^{z}_{x_i}{}^{c} E^{z}_{x_j}{}^{c} = (\pi \ell)^2 [(-1)^i \; i \frac{\cos\theta}{\theta^2 - (\frac{i\pi}{2})^2} \cos\phi \; (-1)^j \; j \frac{\cos\theta}{\theta^2 - (\frac{j\pi}{2})^2} \cos\phi]$$

where $\theta = \xi_k \ell/2$ and $\phi = \xi_k (\frac{\ell + t}{2})$

and $(\pi\ell)^2$ factors out of each element of the resulting matrix of inner product terms.

For the odd mode

$$E^{z_s}_{x_i} E^{z_c}_{x_j} = (\pi\ell)^2[(-1)^i(i)\ \frac{\sin\theta}{\theta^2-(\frac{i}{2}\pi)^2}\ \cos\phi\ (-1)^j\ j\ \frac{\cos\theta}{\theta^2-(\frac{j}{2})^2}\ \sin\phi]\ \overbrace{(-j)(j)}^{+1}$$

$$E^{z_s}_{x_i} E^{z_s}_{x_j} = (\pi\ell)^2[(-1)^i(i)\ \frac{\sin\theta}{\theta^2-(\frac{i\pi}{2})^2}\ \cos\phi\ (-1)^j(j)\ \frac{\sin\theta}{\theta^2-(\frac{i\pi}{2})^2}\ \cos\phi]\ \overbrace{(-j)(-j)}^{-1}$$

$$E^{z_c}_{x_i} E^{z_c}_{x_j} = (\pi\ell)^2[(-1)^i\ i\ \frac{\cos\theta}{\theta^2-(\frac{i\pi}{2})^2}\ \cos\phi\ (-1)^j\ j\ \frac{\cos\theta}{\theta^2-(\frac{j\pi}{2})^2}\ \cos\phi]\ \overbrace{(j)(j)}^{-1}$$

where $\theta = \xi_k\ell/2$ and $\phi = \xi_k(\frac{\ell+t}{2})$

and $(\pi\ell)^2$ factors out of each element of the resulting matrix of the inner product terms. Note, each element of the matrix will be multiplied by $(-1)^{i+j+1}$ for the odd mode.

The singularities for the $E_x^x(\alpha_n)$ and $E_x^{z_c}(\xi_k)$ are the same as in the single cavity case. The singularity for $E_x^{z_s}(\xi_k)$ is found by

$$\lim_{\theta \to \frac{q\pi}{2}} \frac{\sin\theta}{\theta^2 - (\frac{q\pi}{2})^2} = \frac{(-1)^q}{q\pi} \quad , \text{ for q even.}$$

The inner product is defined the same as the single cavity case.

N_{max} and K_{max} are found in the same manner as in the single cavity case. N_{max} remains unchanged. K_{max} becomes

$$K_{max} = \frac{1}{2} + (2 + t/\ell_e)[zconst + (\frac{q}{2})^2]^{1/2} \quad , \text{ even mode}$$

$$K_{max} = (2 + t/\ell_o)[zconst + (\frac{q}{2})^2]^{1/2} \quad , \text{ odd mode}$$

APPENDIX D

NUMERICAL RESULTS FOR THE SINGLE AND COUPLED RESONANT CAVITIES

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THIS IS THE FINSTRP PROGRAM TO DETERMINE THE ODD AND
EVEN MODE RESONANT LENGTHS OF TWO UNILATERALLY COUPLED
FIN-LINE RESONANT CAVITIES, THE CORRESPONDING EQUIVALENT
CIRCUIT REACTANCES, AND THE SCATTERING PARAMETERS OF THE
INDUCTIVE STRIP USING A BASIS SET OF SIN AND COS FUNCTIONS.
THE DIMENSION OF THE MATRIX IS =    12
RESIDUE (ACCURACY) = 0.0001

FINLINE PARAMETERS FOR THE X-BAND WAVEGUIDE (8.0 TO 12.0 GHz)
ARE:
EPSR1 =  1.0        EPSR2 =  1.0        EPSR3 =  1.0
H1/D =  4.5         H2/D  =  3.5        B/D =  4.0
```

W/B	W/D	D/L	LPR/L
1.000	4.000	0.0677	1.747

TOVD	TOVW	LE/LPR	LO/LPR	XS
0.050	0.012	0.373	0.500	0.511
0.100	0.025	0.380	0.500	0.472
0.200	0.050	0.389	0.500	0.419
0.500	0.125	0.409	0.500	0.321
1.000	0.250	0.425	0.487	0.295
2.000	0.500	0.444	0.500	0.185
5.000	1.250	0.458	0.464	0.251
10.000	2.500	0.460	0.460	0.255
16.000	4.000	0.460	0.460	0.256

W/B	W/D	D/L	LPR/L
1.000	4.000	0.0720	1.574

TOVD	TOVW	LE/LPR	LO/LPR	XS
0.050	0.012	0.355	0.500	0.645
0.100	0.025	0.361	0.500	0.595
0.200	0.050	0.371	0.500	0.529
0.500	0.125	0.390	0.491	0.443
1.000	0.250	0.409	0.487	0.360
2.000	0.500	0.430	0.470	0.332
5.000	1.250	0.447	0.455	0.318
10.000	2.500	0.451	0.451	0.318
16.000	4.000	0.451	0.451	0.317

W/B	W/D	D/L	LPR/L
1.000	4.000	0.0762	1.461

TOVD	TOVW	LE/LPR	LO/LPR	XS
0.050	0.012	0.342	0.500	0.766
0.100	0.025	0.348	0.498	0.709
0.200	0.050	0.358	0.497	0.634
0.500	0.125	0.377	0.489	0.523
1.000	0.250	0.396	0.480	0.446
2.000	0.500	0.417	0.466	0.396
5.000	1.250	0.438	0.448	0.375
10.000	2.500	0.443	0.443	0.374
16.000	4.000	0.443	0.443	0.374

W/B	W/D	D/L	LPR/L
1.000	4.000	0.0804	1.382

TOVD	TOVW	LE/LPR	LO/LPR	XS
0.050	0.012	0.332	0.499	0.882
0.100	0.025	0.338	0.498	0.816
0.200	0.050	0.347	0.495	0.731
0.500	0.125	0.365	0.487	0.604
1.000	0.250	0.384	0.477	0.517
2.000	0.500	0.406	0.462	0.459
5.000	1.250	0.429	0.442	0.433
10.000	2.500	0.435	0.436	0.432
16.000	4.000	0.435	0.435	0.432

W/B	W/D	D/L
1.000	4.000	0.0847

1.325

TOVD	TOVW	LE/LPR	LC/LPR	XS
0.050	0.012	0.324	0.499	1.004
0.100	0.025	0.329	0.497	0.929
0.200	0.050	0.338	0.494	0.834
0.500	0.125	0.355	0.486	0.691
1.000	0.250	0.372	0.474	0.599
2.000	0.500	0.394	0.457	0.533
5.000	1.250	0.418	0.434	0.502
10.000	2.500	0.425	0.427	0.499
16.000	4.000	0.426	0.426	0.500

1.000 4.000 0.0889

1.280

TOVD	TOVW	LE/LPR	LC/LPR	XS
0.050	0.012	0.317	0.499	1.115
0.100	0.025	0.322	0.497	1.032
0.200	0.050	0.330	0.494	0.926
0.500	0.125	0.346	0.485	0.777
1.000	0.250	0.362	0.473	0.676
2.000	0.500	0.382	0.454	0.608
5.000	1.250	0.408	0.428	0.572
10.000	2.500	0.417	0.419	0.569
16.000	4.000	0.418	0.418	0.568

1.000 4.000 0.0931

1.246

TOVD	TOVW	LE/LPR	LO/LPR	XS
0.050	0.012	0.311	0.498	1.236
0.100	0.025	0.316	0.496	1.145
0.200	0.050	0.323	0.493	1.032
0.500	0.125	0.337	0.483	0.874
1.000	0.250	0.351	0.470	0.773
2.000	0.500	0.369	0.449	0.701
5.000	1.250	0.395	0.421	0.659
10.000	2.500	0.406	0.409	0.656
16.000	4.000	0.408	0.408	0.655

1.000 4.000 0.0974

1.217

TOVD	TOVW	LE/LPR	LO/LPR	XS
0.050	0.012	0.307	0.498	1.353
0.100	0.025	0.311	0.496	1.256
0.200	0.050	0.317	0.492	1.135
0.500	0.125	0.329	0.482	0.977
1.000	0.250	0.342	0.467	0.875
2.000	0.500	0.357	0.445	0.806
5.000	1.250	0.381	0.413	0.764
10.000	2.500	0.394	0.400	0.758
16.000	4.000	0.396	0.397	0.758

1.000 4.000 0.1016

1.195

TOVD	TOVW	LE/LPR	LO/LPR	XS
0.050	0.012	0.302	0.498	1.475
0.100	0.025	0.306	0.496	1.371
0.200	0.050	0.312	0.492	1.248
0.500	0.125	0.322	0.481	1.090
1.000	0.250	0.332	0.465	0.998
2.000	0.500	0.344	0.442	0.938
5.000	1.250	0.365	0.406	0.902
10.000	2.500	0.379	0.389	0.896
16.000	4.000	0.383	0.385	0.895

Figure D-1 Odd and Even Mode Resonant Lengths for the Coupled Fin-Line Resonant Cavities for w/b=1.0, t/D=0.05, 0.1, 0.2, 0.5, 1.0, 2.0, 5.0, 10.0, and 16.0, D=0.1 inch, ε_{r_2}=1.0, and for 8 through 12 GHz

THIS IS THE FINCAV PROGRAM TO DETERMINE THE RESONANT LENGTH OF A SINGLE FIN-LINE RESONANT CAVITY, THE GUIDE WAVELENGTH, AND THE EQUIVALENT REACTANCE OF THE SHORTING SEPTUM (INDUCTIVE DISCONTINUITY)
THE DIMENSION OF THE MATRIX IS = 12
RESIDUE (ACCURACY) = 0.0001

FINLINE PARAMETERS FOR THE X-BAND WAVEGUIDE (8.0 TO 12.0 GHz) ARE:

EPSR1 = 1.0 EPSR2 = 1.0 EPSR3 = 1.0
H1/D = 4.5 H2/D = 3.5 B/D = 4.0

W/B	W/D	D/L	LPR/L	TOVD	LRES/LPR
1.000	4.000	0.0677			
			1.747		
				0.200	0.493
				0.500	0.483
				1.000	0.469
				2.000	0.449
				5.000	0.424
				10.000	0.418
				16.000	0.417
1.000	4.000	0.0720			
			1.574		
				0.200	0.491
				0.500	0.479
				1.000	0.462
				2.000	0.438
				5.000	0.408
				10.000	0.400
				16.000	0.400
1.000	4.000	0.0762			
			1.461		
				0.200	0.490
				0.500	0.477
				1.000	0.457
				2.000	0.429
				5.000	0.394
				10.000	0.384
				16.000	0.383
1.000	4.000	0.0804			
			1.382		
				0.200	0.489
				0.500	0.474
				1.000	0.453
				2.000	0.421
				5.000	0.380
				10.000	0.368
				16.000	0.367
1.000	4.000	0.0847			
			1.325		
				0.200	0.488
				0.500	0.471
				1.000	0.448
				2.000	0.413
				5.000	0.366
				10.000	0.351
				16.000	0.350
1.000	4.000	0.0889			
			1.280		
				0.200	0.487
				0.500	0.469
				1.000	0.443
				2.000	0.405
				5.000	0.353
				10.000	0.334
				16.000	0.332

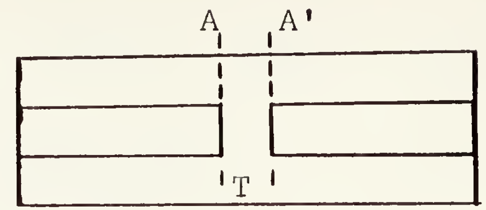
1.000	4.000	0.0931			
			1.246		
				0.200	0.486
				0.500	0.466
				1.000	0.438
				2.000	0.397
				5.000	0.338
				10.000	0.315
				16.000	0.312
1.000	4.000	0.0974			
			1.217		
				0.200	0.485
				0.500	0.464
				1.000	0.434
				2.000	0.389
				5.000	0.324
				10.000	0.295
				16.000	0.289
1.000	4.000	0.1016			
			1.195		
				0.200	0.483
				0.500	0.461
				1.000	0.430
				2.000	0.382
				5.000	0.308
				10.000	0.272
				16.000	0.262

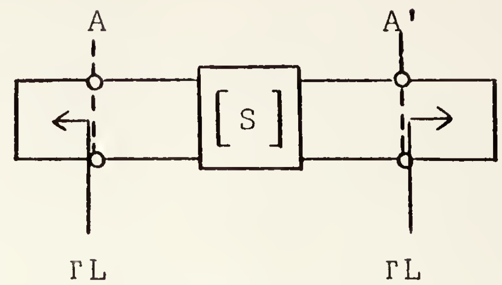
Figure D-2 Resonant Lengths for the Single Fin-Line Resonant Cavity for w/b=1.0, t/D=0.2, 0.5, 1.0, 2.0, 5.0, 10.0, and 16.0, D=0.1 inch, ε_{r_2}=1.0, and for 8 through 12 GHz

APPENDIX E
THEORETICAL ANALYSIS OF THE SCATTERING PARAMETERS OF THE FIN-LINE INDUCTIVE STRIP

Solution of Inductive Discontinuity

Using S Parameters





This network is a) lossless

b) symmetric with respect to reference planes A and A'

c) reciprocal

As a result the scattering matrix of the network is a) unitary

b) $S_{11} = S_{22}$

c) symmetric ($S_{21} = S_{12}$)

Therefore

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{11} \end{bmatrix} \qquad \begin{matrix} S_{11}S_{11}^* + S_{12}S_{12}^* = 1 \\ S_{11}S_{12}^* + S_{12}S_{11}^* = 0 \end{matrix}$$

Hence $\quad |S_{12}| = \sqrt{1 - S_{11}{}^2}$

$$\theta_{12} = \theta_{11} - \pi/2, \text{ since the strip is inductive}$$

Γ_L is the reflection coefficient looking into the shorted fin-line at A or A'

$$\Gamma_L = -1\ e^{-j2\beta\ell}$$

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$\text{and } a_1 = \Gamma_L b_1,\ a_2 = \Gamma_L b_2$$

$$b_2 = S_{12}a_1 + S_{11}a_2$$

Substituting: $b_1 = S_{11}\Gamma_L b_1 + S_{12}\Gamma_L b_2$

$$b_2 = S_{12}\Gamma_L b_1 + S_{11}\Gamma_L b_2$$

$$(1-S_{11}\Gamma_L)b_1 - (S_{12}\Gamma_L)b_2 = 0$$

$$-(S_{12}\Gamma_L)b_1 + (1-S_{11}\Gamma_L)b_2 = 0$$

elliminating the b's:

$$(1-S_{11}\Gamma_L)^2 + (S_{12}\Gamma_L)^2 = 0$$

$$(S_{11}^2 - S_{12}^2)\Gamma_L^2 - 2S_{11}\Gamma_L + 1 = 0$$

Solving for Γ_L with the quadratic formula

$$\Gamma_L = \frac{2\,S_{11} \pm \sqrt{(2S_{11})^2 - 4(S_{11}^2 - S_{12})^2}}{2(S_{11} + S_{12})(S_{11} - S_{12})}$$

giving $\Gamma_L = \dfrac{S_{11} \pm S_{12}}{(S_{11} + S_{12})(S_{11} - S_{12})}$

and $$\left.\begin{aligned} \Gamma_{Le} &= \frac{1}{S_{11} - S_{12}} \\ \Gamma_{Lo} &= \frac{1}{S_{11} + S_{12}} \end{aligned}\right\} \qquad \begin{aligned} S_{11} &= \frac{1}{2}\left(\frac{\Gamma_e + \Gamma_o}{\Gamma_e \Gamma_o}\right) \\ S_{12} &= \frac{1}{2}\left(\frac{\Gamma_e - \Gamma_o}{\Gamma_e \Gamma_o}\right) \end{aligned}$$

$$\Gamma_{Le} = - e^{-j4\pi \ell e/\lambda'} \quad \text{and} \quad \Gamma_{Lo} = -e^{-j4\pi \ell o/\lambda'} .$$

Substituting,

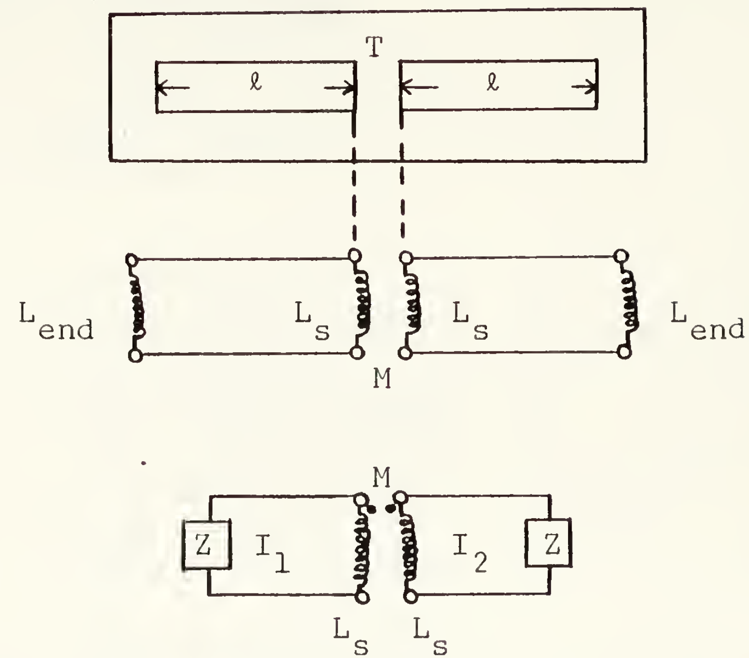
$$S_{11} = \frac{1}{2}\left[\frac{-e^{-j2\beta\ell e} - e^{-j2\beta\ell o}}{e^{-j2\beta(\ell e+\ell o)}}\right]$$

$$= -e^{j2\pi(\ell e/\lambda' + \ell o/\lambda')} \cos 2\pi(\ell e/\lambda' - \ell o/\lambda')$$

$$S_{12} = \frac{1}{2}\left[\frac{-e^{-j2\beta\ell e} + e^{-j2\beta\ell o}}{e^{-j2\beta(\ell e+\ell o)}}\right]$$

$$= j\, e^{j2\pi(\ell e/\lambda' + \ell o/\lambda')} \sin 2\pi(\ell e/\lambda' - \ell o/\lambda').$$

From Knorr, notes for reference 7, the equivalent circuit reactances are determined as follows:



$$x_{end} = \omega L_{end}$$

$$Z_L = j\omega L_s$$

$$Z_M = j\omega M$$

$$Z = Z_o \frac{Z_R + j\, Z_o \tan \beta\ell}{Z_o + j\, Z_R \tan \beta\ell}$$

$$x_{end} = \tan(\tan^{-1} x_{end})$$

$$z = \frac{j\, x_{end} + j \tan \beta\ell}{1 - x_{end} \tan \beta\ell}$$

$$\frac{Z_L}{Z_o} = jX_s = j\omega \frac{L_s}{Z_o}$$

$$z = j\,\frac{\tan(\tan^{-1}x_{end}) + \tan\beta\ell}{1 - \tan(\tan^{-1}X_{end})\tan\beta\ell}$$

Noting $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$$z = j\tan(\beta\ell + \tan^{-1}X_{end}) = jx$$

At resonance:
$$zI_1 + j\omega L_s I_1 + j\omega m I_2 = 0$$
$$zI_2 + j\omega L_s I_2 + j\omega m I_1 = 0$$

Normalizing
$$jx_s I_1 + jmI_2 = jxI_1$$
$$jmI_1 + jx_s I_2 = jxI_2$$
$$\overline{}$$
$$(x_s+m)(I_1+I_2) = x(I_1+I_2)$$

Hence $x = x_s \pm m$ yielding $\begin{cases} x_{even} = x_s + m \\ X_{odd} = X_s - M \end{cases}$

Solving for x_s and m;

$$x_s = \frac{1}{2}(x_{even} + x_{odd})$$

$$m = \frac{1}{2}(x_{even} - x_{odd})$$

and

$$x_s = \frac{1}{2}[\tan(2\pi\ \ell e/\lambda' + \tan^{-1}x_{end}) + \tan(2\pi\ \ell o/\lambda' + \tan^{-1}x_{end})]$$

$$= \frac{1}{2}[\tan(\pi - 2\pi\ \ell e/\lambda') + \tan(\pi - 2\pi \ell o/\lambda')]$$

$$m = \frac{1}{2}[\tan(\pi - 2\pi\ \ell e/\lambda') - \tan(\pi - 2\pi\ \ell o/\lambda')]$$

APPENDIX F
NUMERICAL RESULTS FOR THE SCATTERING PARAMETERS OF THE FIN-LINE INDUCTIVE STRIP

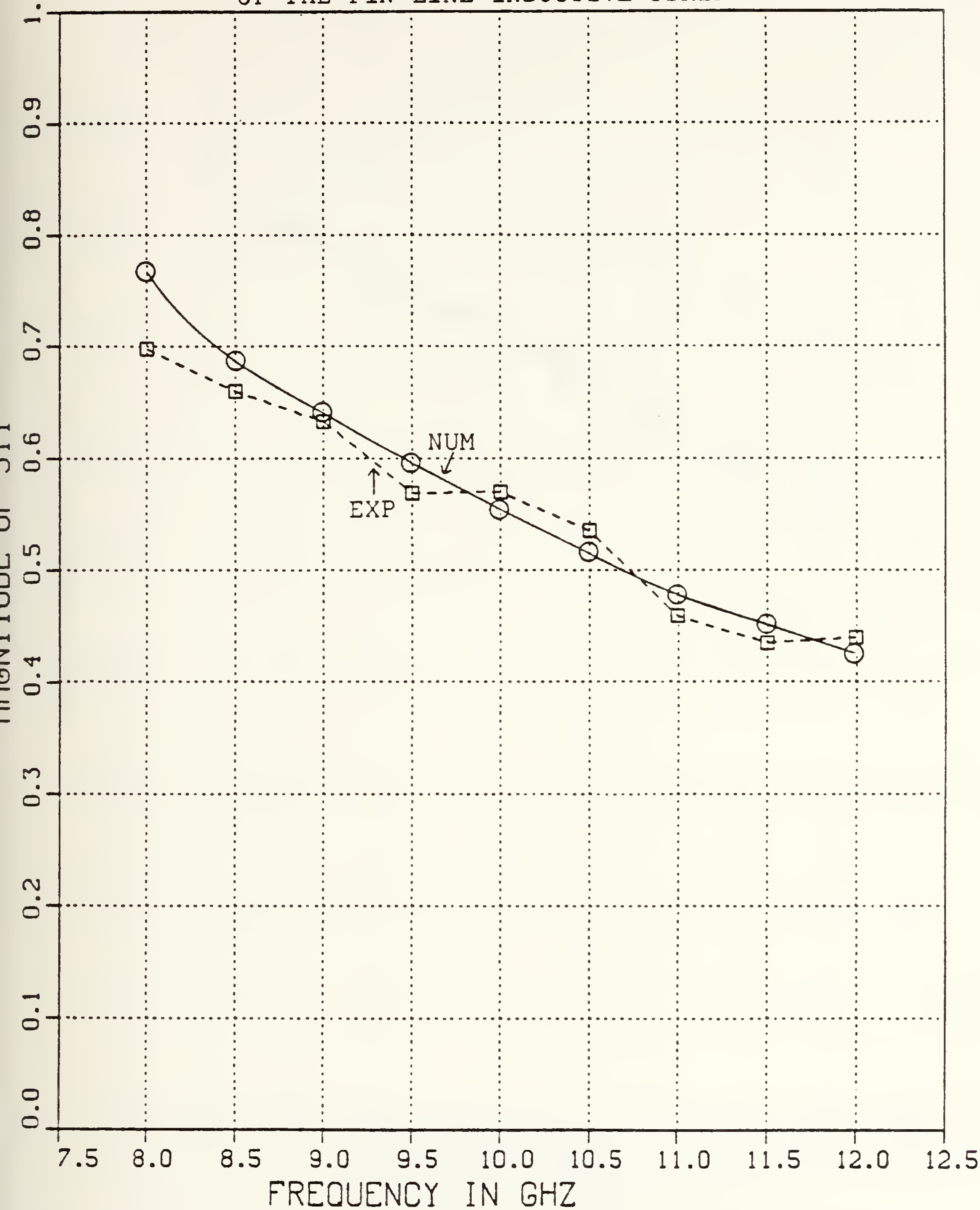


Figure F-1 $|S_{11}|$ v.s. Frequency for T=.02 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

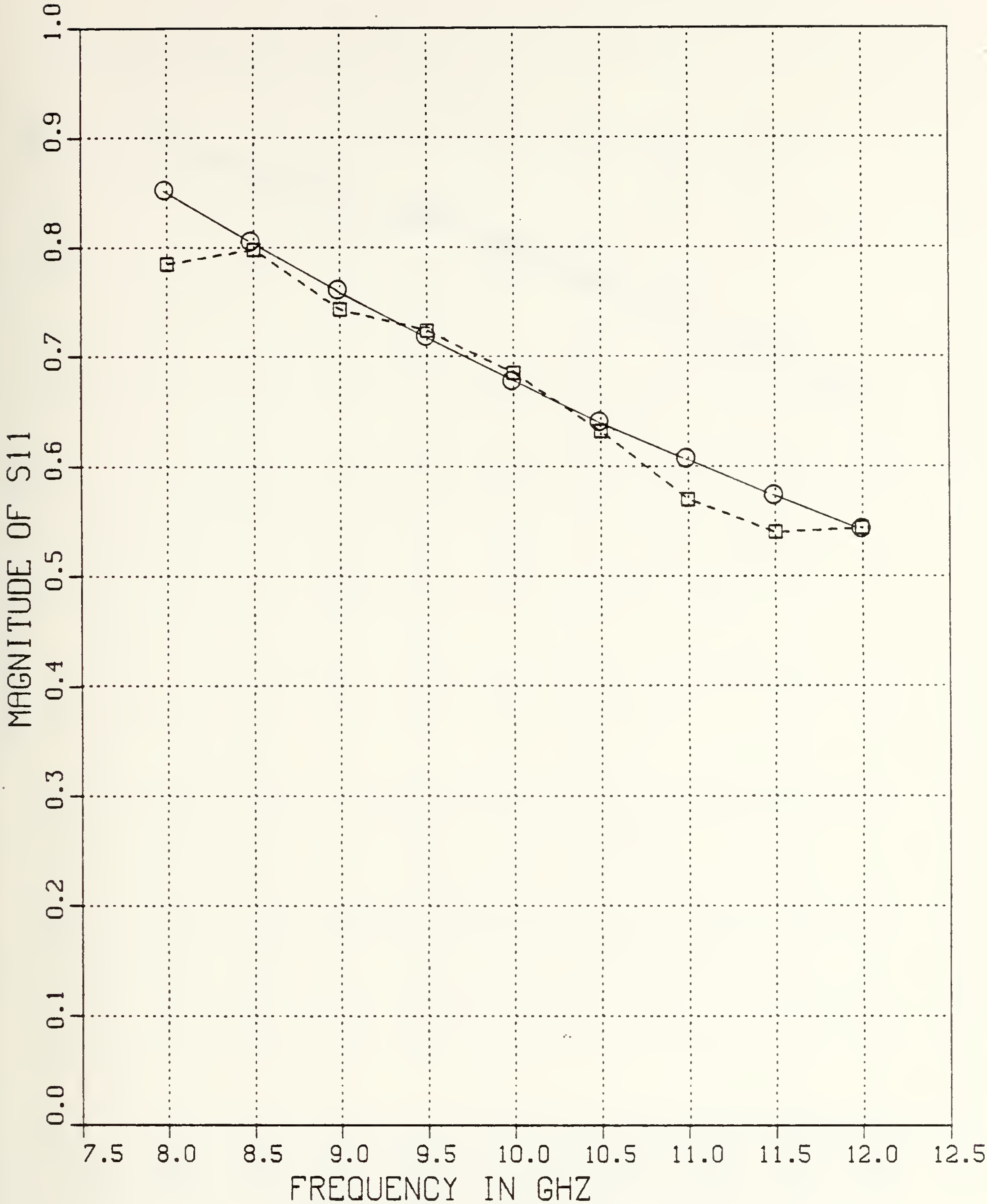


Figure F-2 $|S_{11}|$ v.s. Frequency for T=.05 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

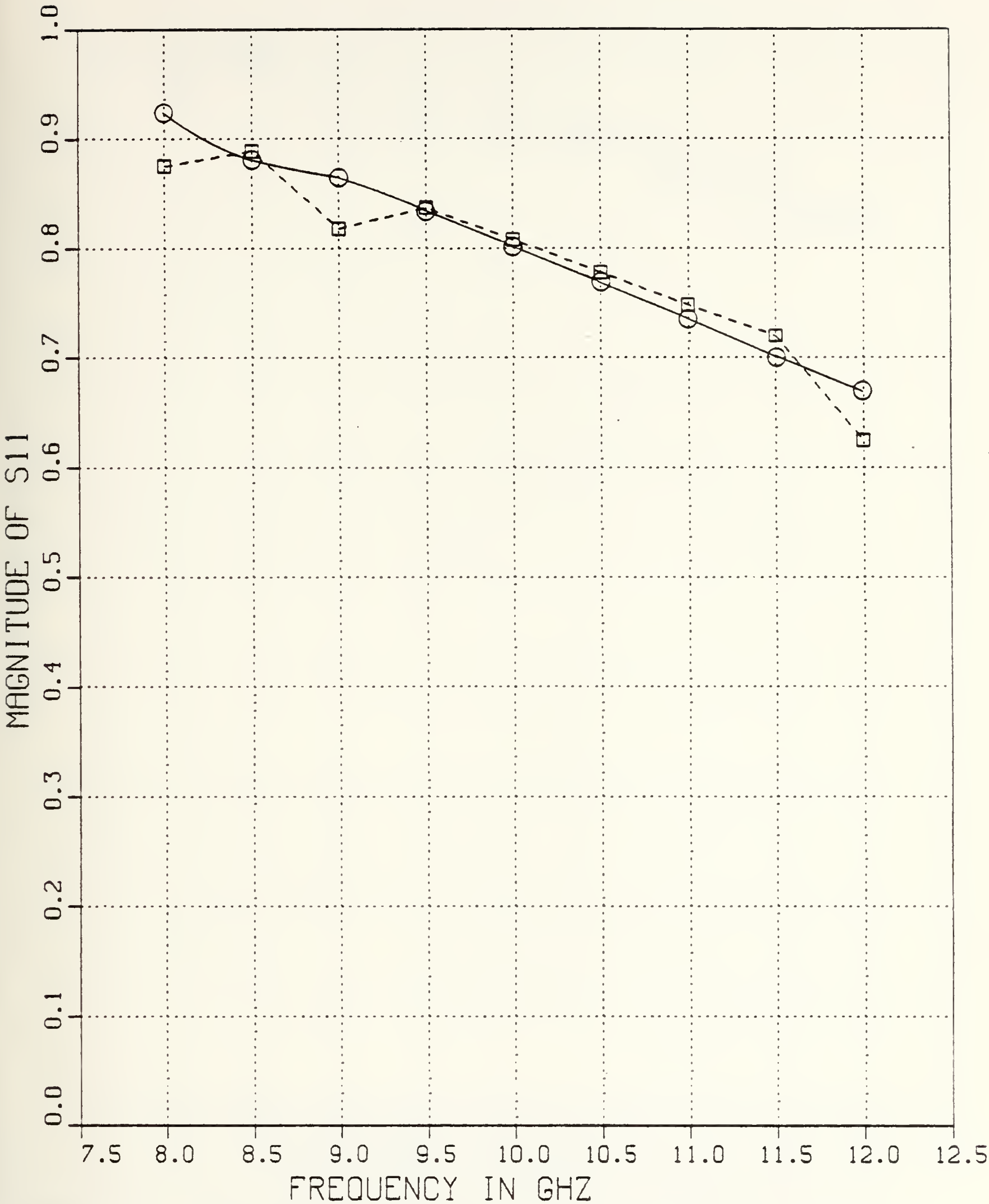


Figure F-3 $|S_{11}|$ v.s. Frequency for T=0.1 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

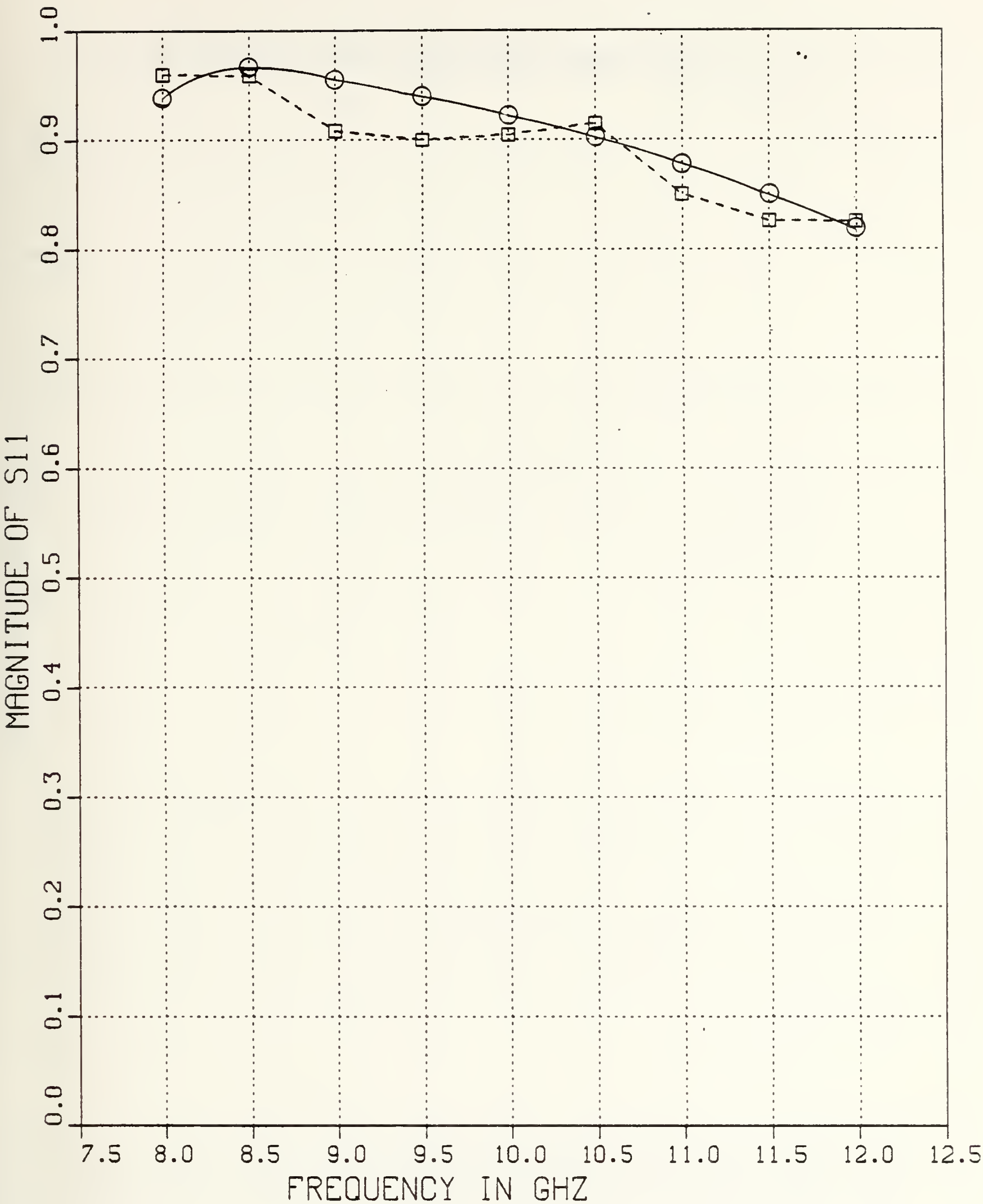


Figure F-4 $|S_{11}|$ v.s. Frequency for T=0.2 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

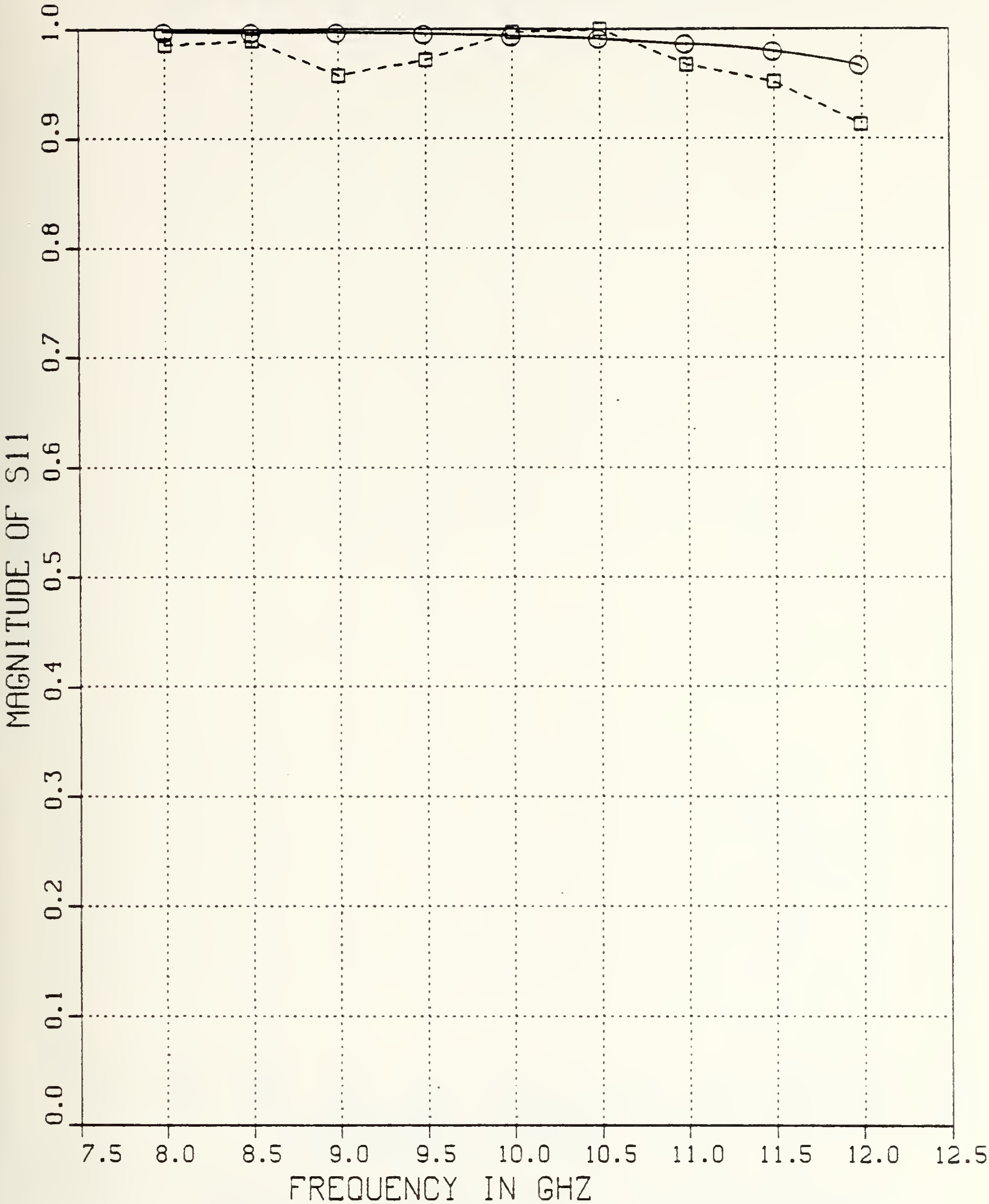


Figure F-5 $|S_{11}|$ v.s. Frequency for T=0.5 inch Inductive Strip, w/b=1.0, and $\varepsilon_{r_2}=1.0$

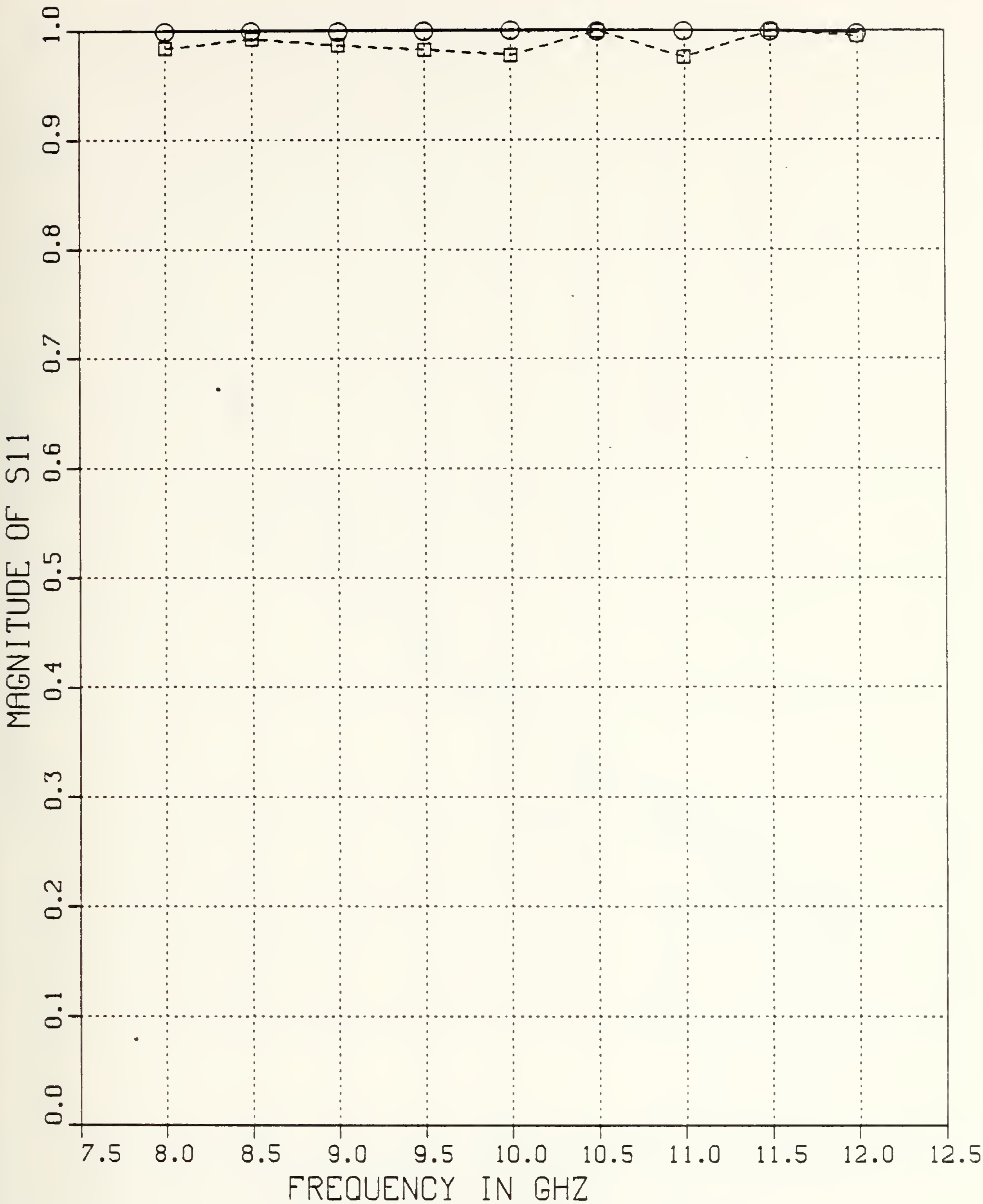
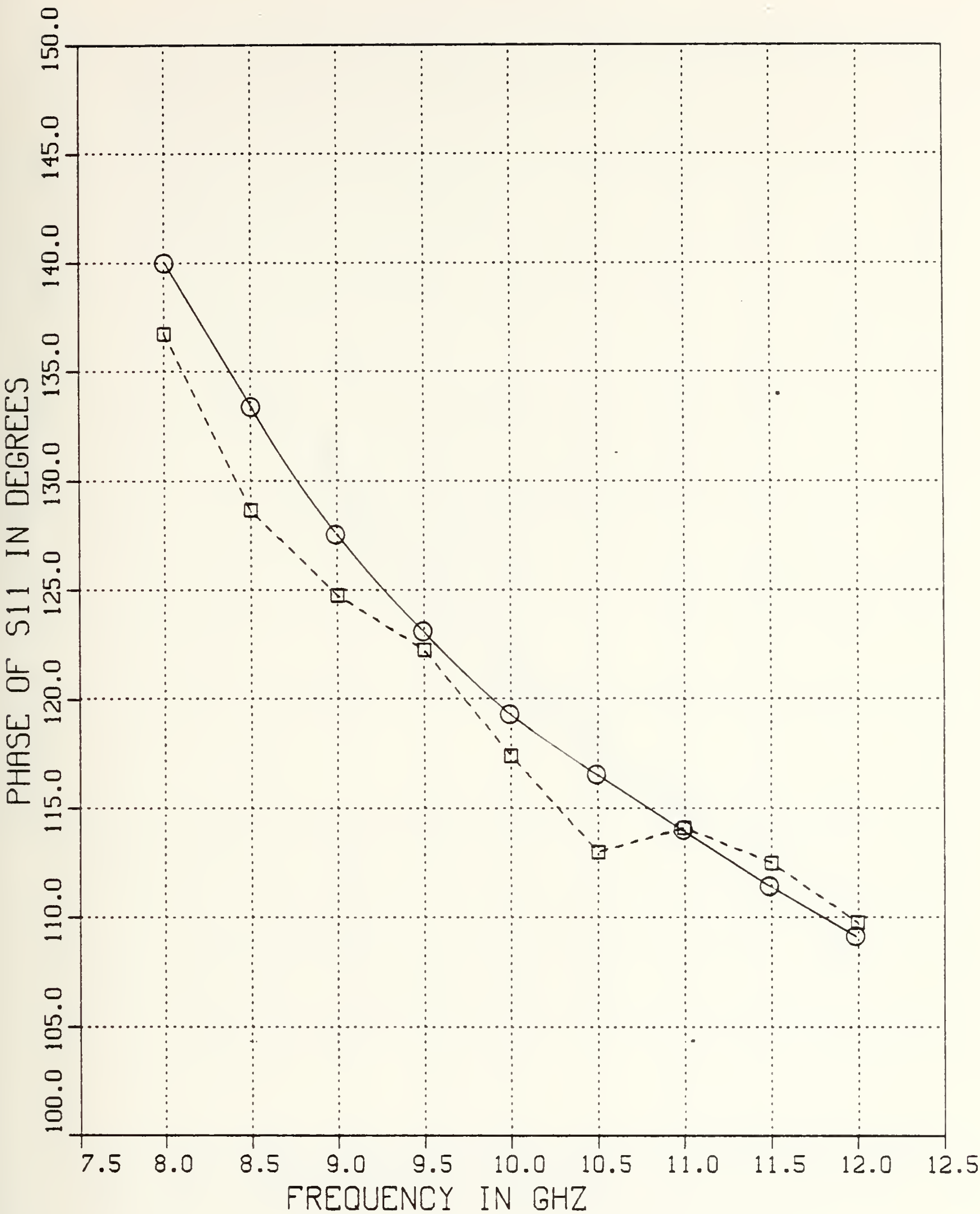


Figure F-6 $|S_{11}|$ v.s. Frequency for T=1.0 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0



Fiugre F-7 θ_{11} v.s. Frequency for T=.02 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

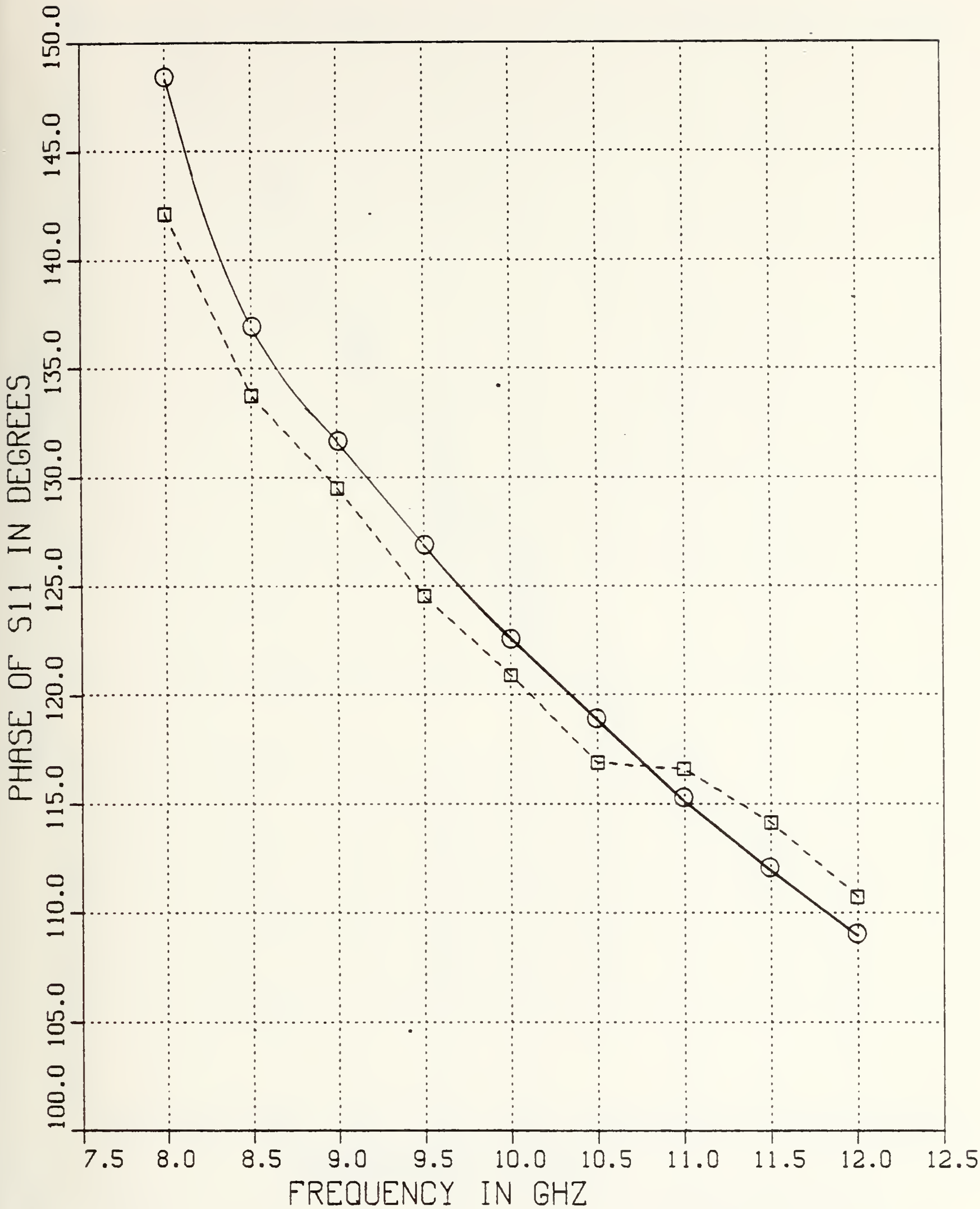


Figure F-8 θ_{11} v.s. Frequency for T=.05 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

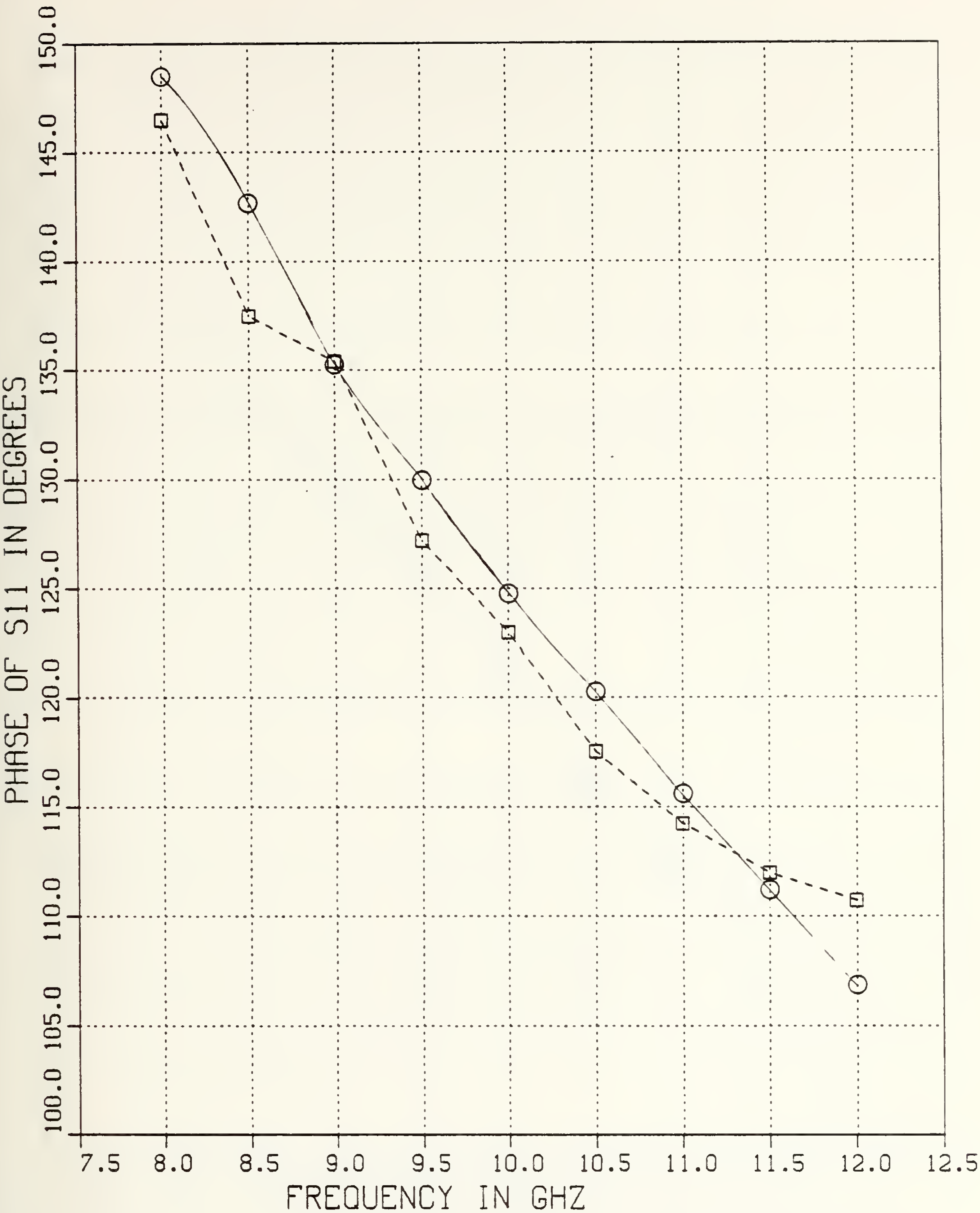


Figure F-9 θ_{11} v.s. Frequency for T=0.1 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

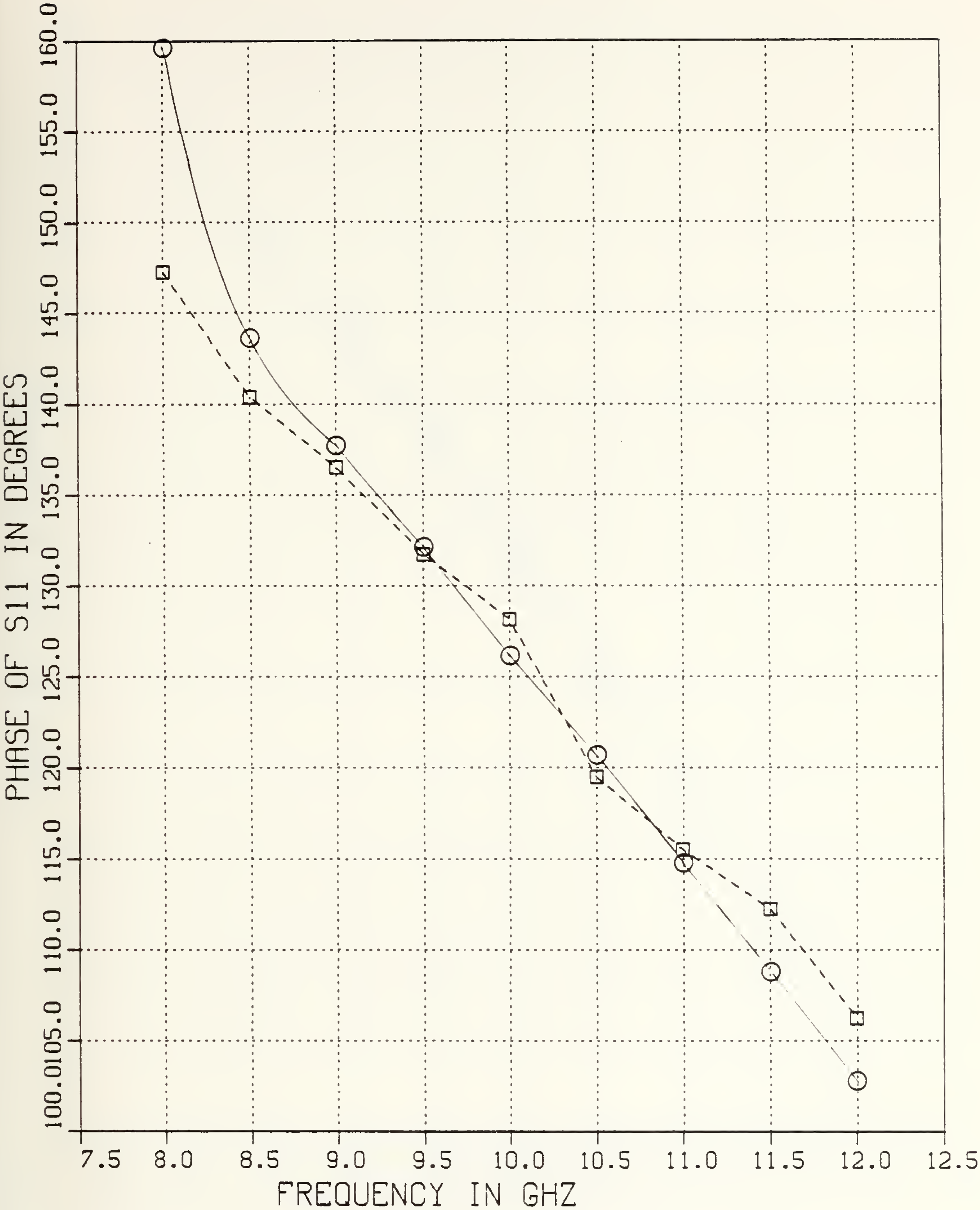


Figure F-10 θ_{11} v.s. Frequency for T=0.2 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

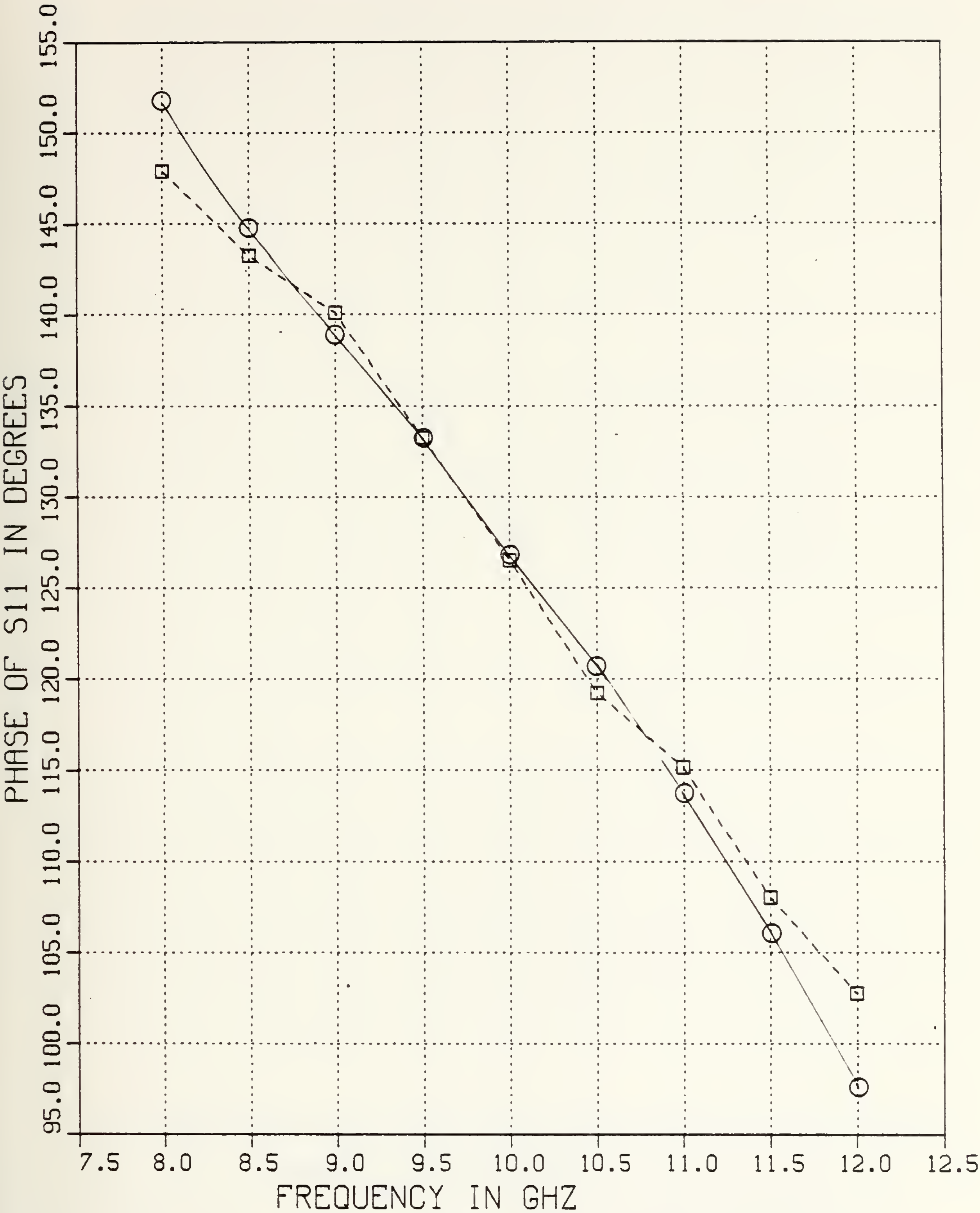


Figure F-11 θ_{11} v.s. Frequency for T=0.5 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

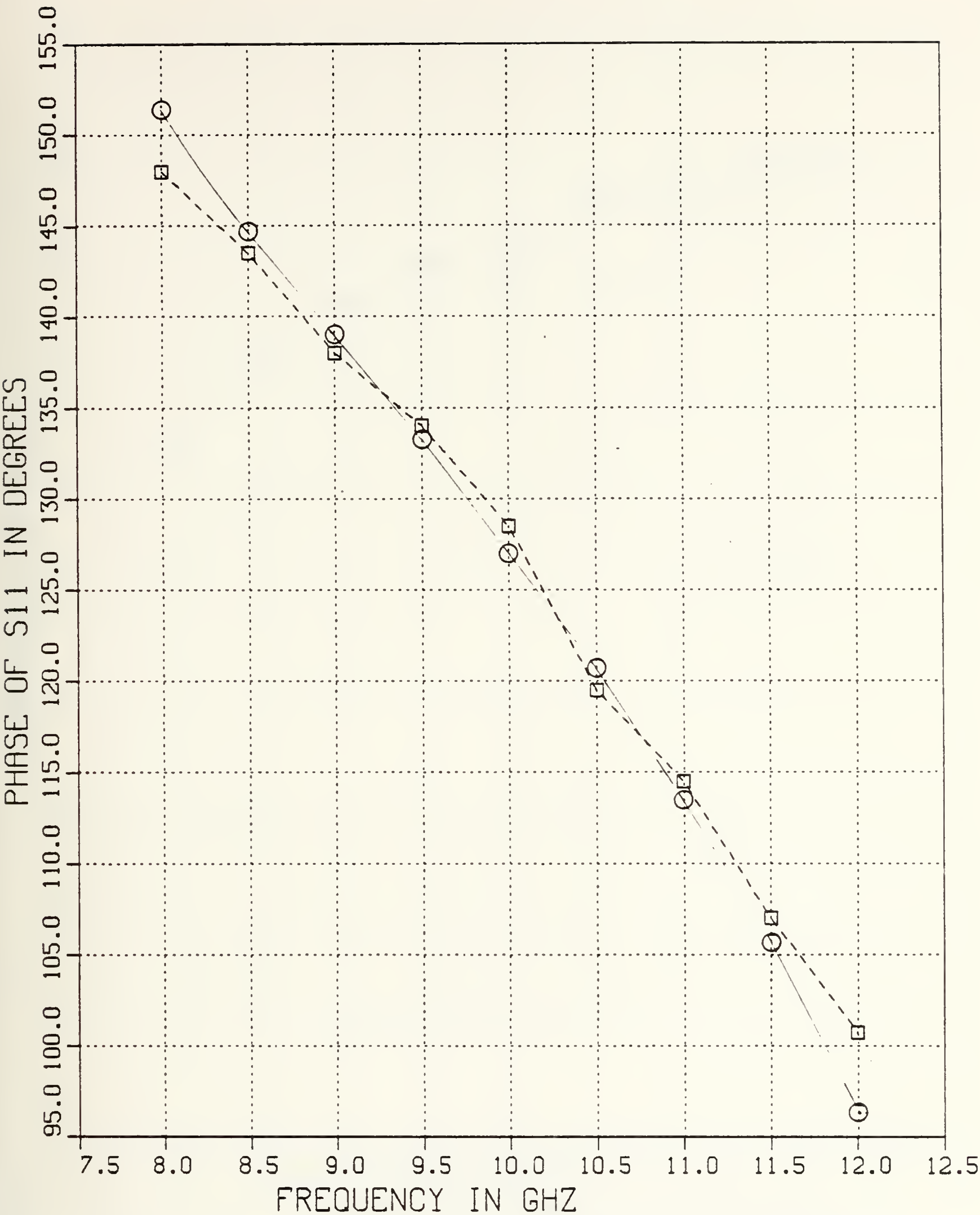


Figure F-12 θ_{11} v.s. Frequency for T=1.0 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

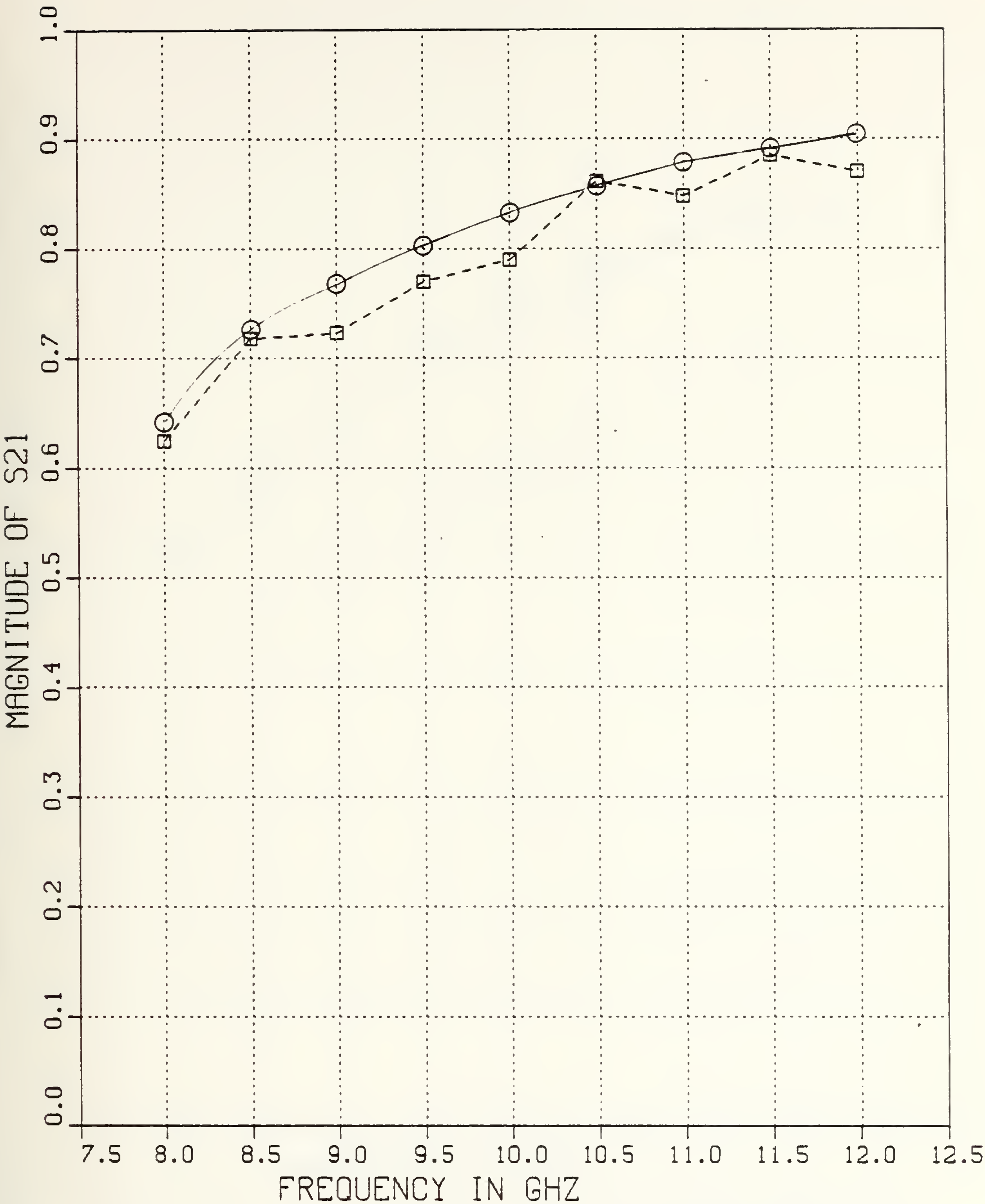


Figure F-13 $|S_{21}|$ v.s. Frequency for T=.02 inch Inductive Strip, w/b/=1.0, and ε_{r_2}=1.0

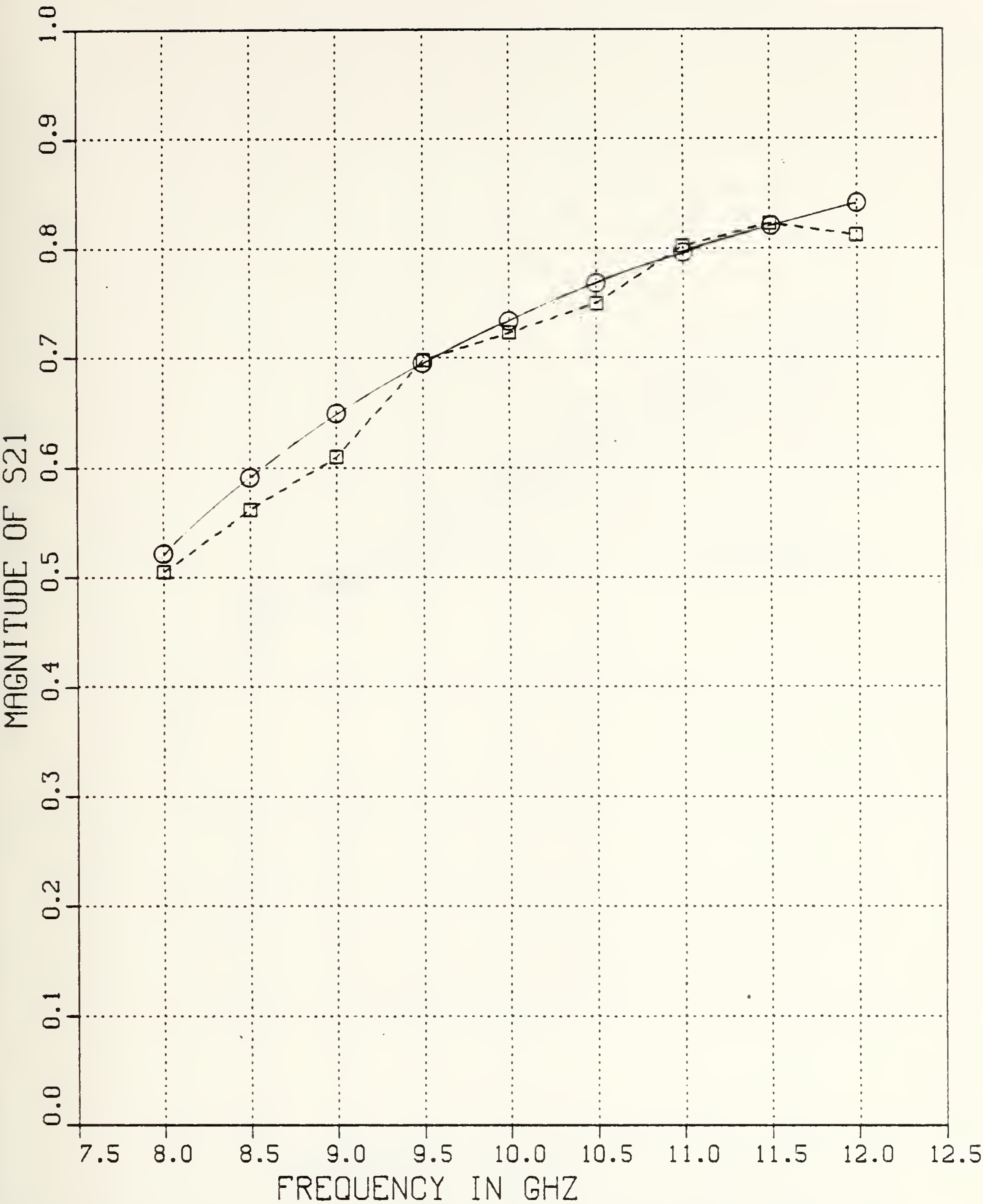


Figure F-14 $|S_{21}|$ v.s. Frequency for T=.05 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

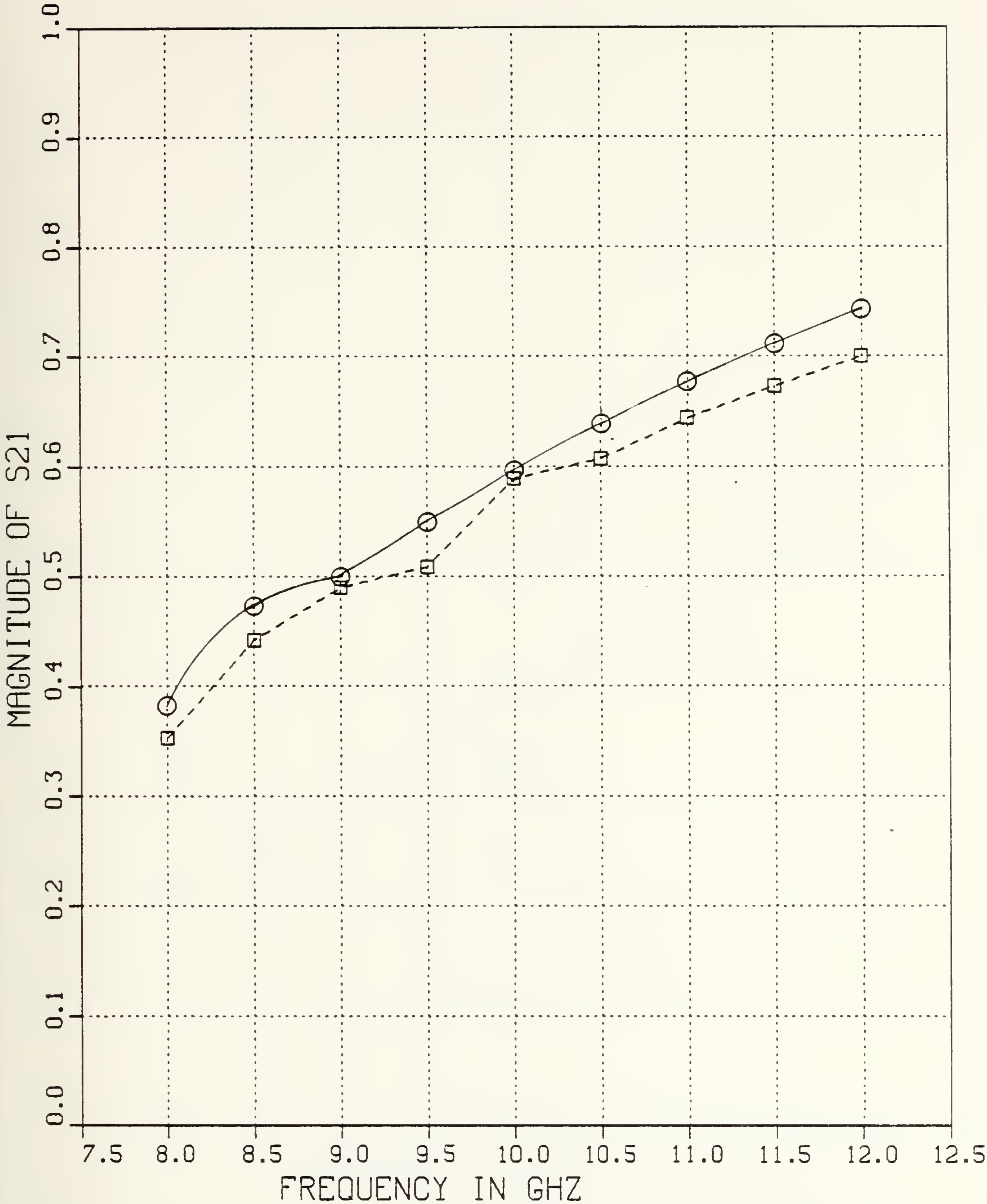


Figure F-15 $|S_{21}|$ v.s. Frequency for T=0.1 inch Inductive Strip, w/b=1.0, and $\varepsilon_{r_2}=1.0$

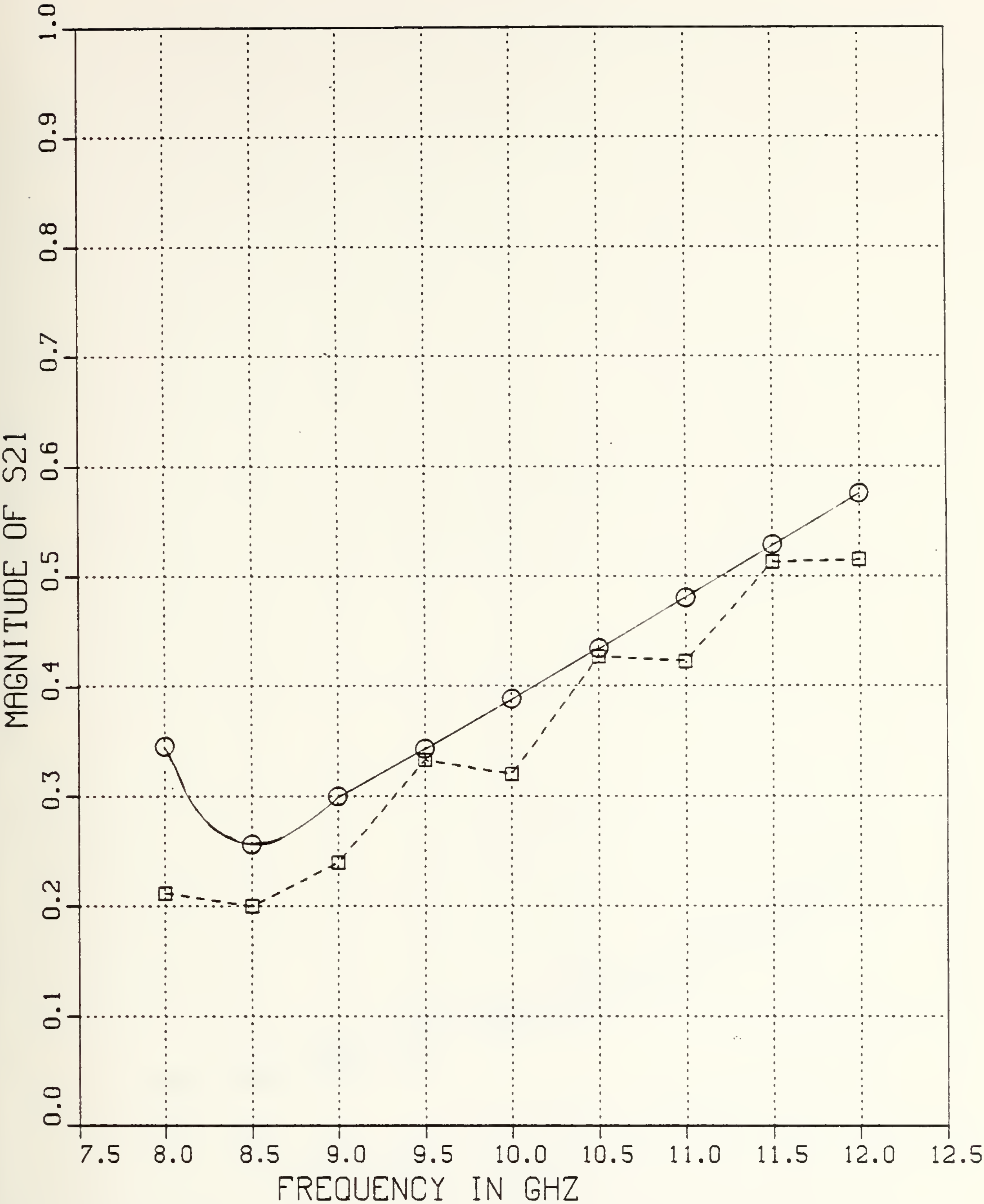


Figure F-16 $|S_{21}|$ v.s. Frequency for T=0.2 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

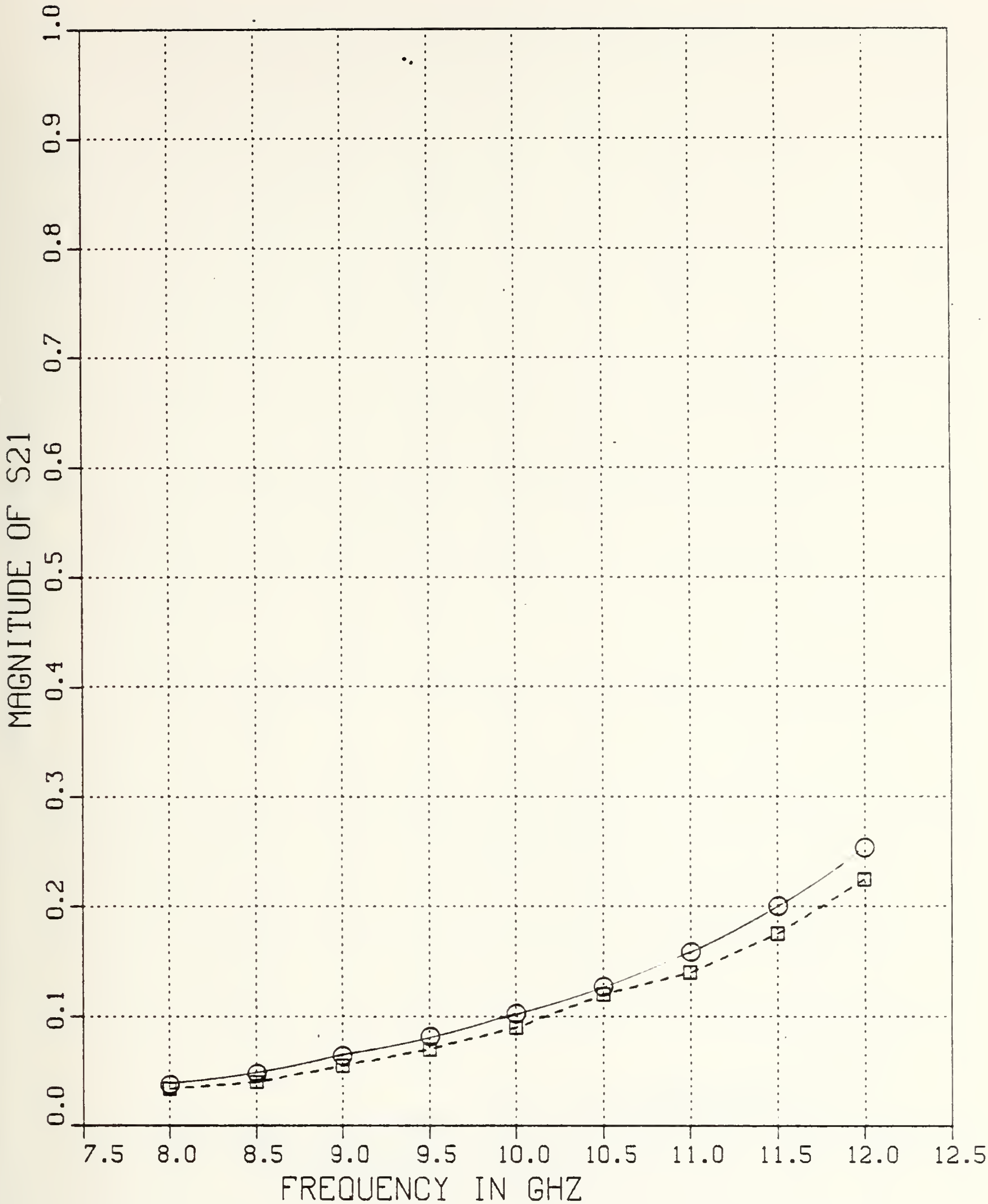


Figure F-17 $|S_{21}|$ v.s. Frequency for T=0.5 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

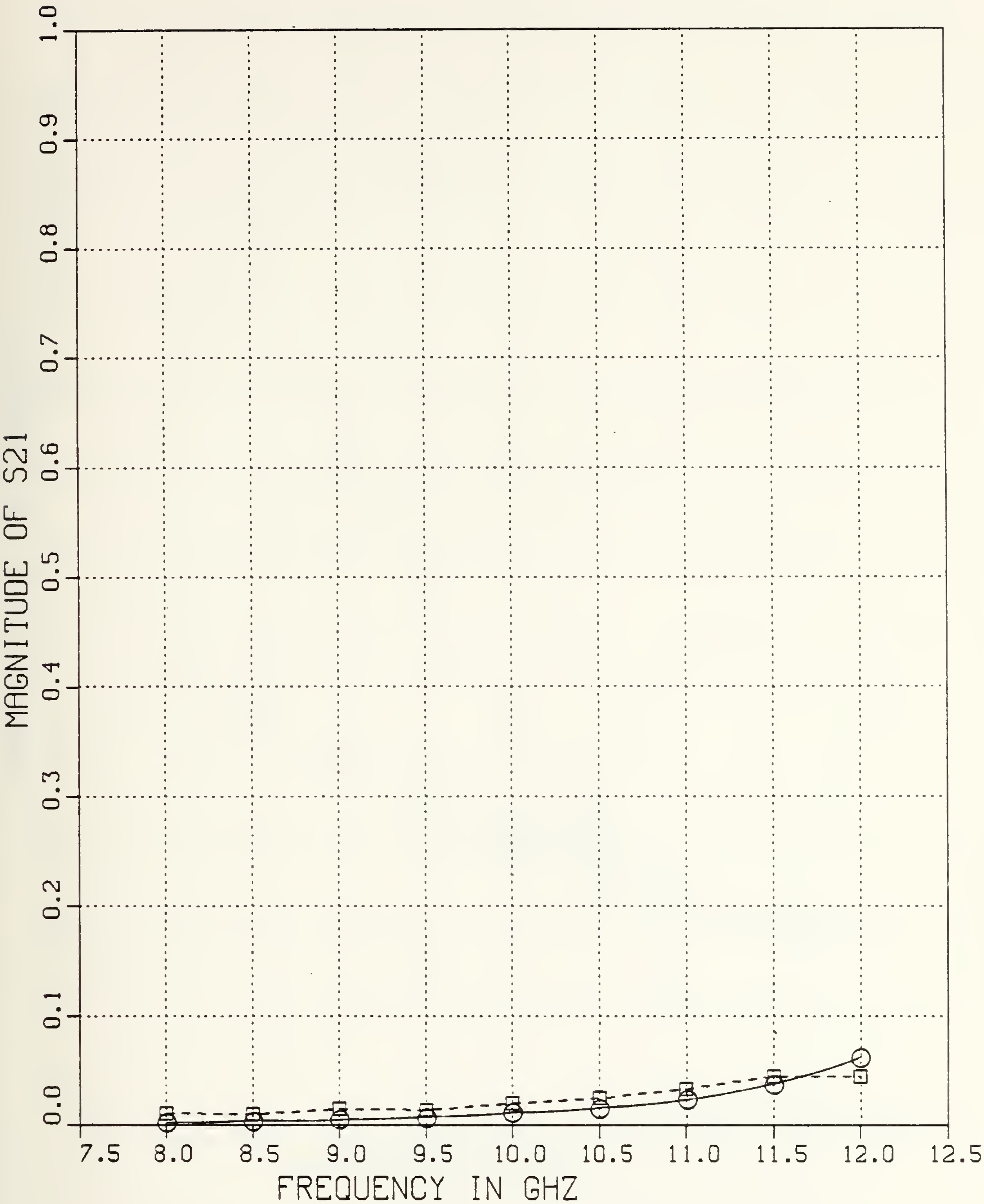


Figure F-18 $|S_{21}|$ v.s. Frequency for T=1.0 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

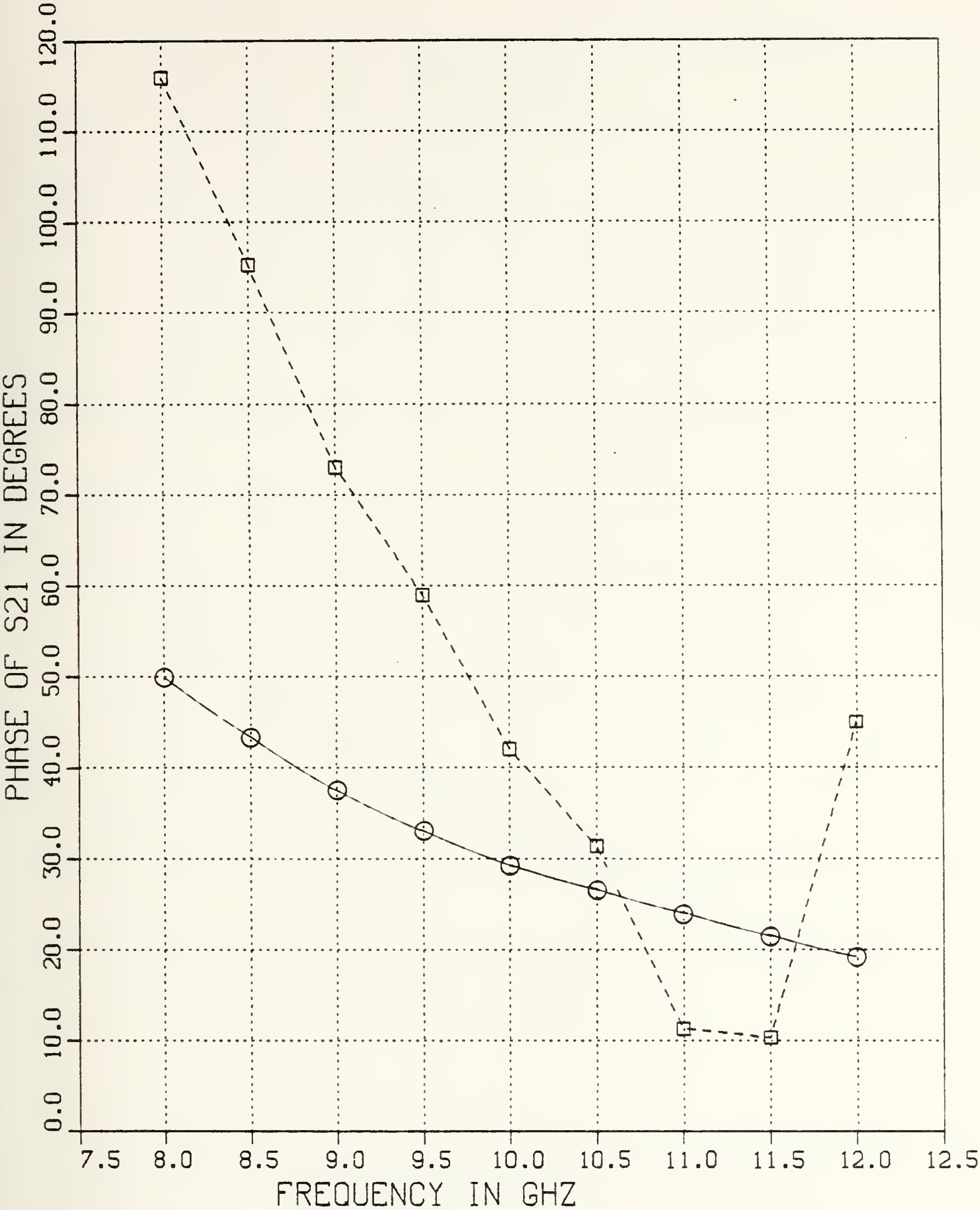


Figure F-19 θ_{21} v.s. Frequency for T=.02 inch Inductive Strip, w/d=1.0, and ε_{r_2}=1.0

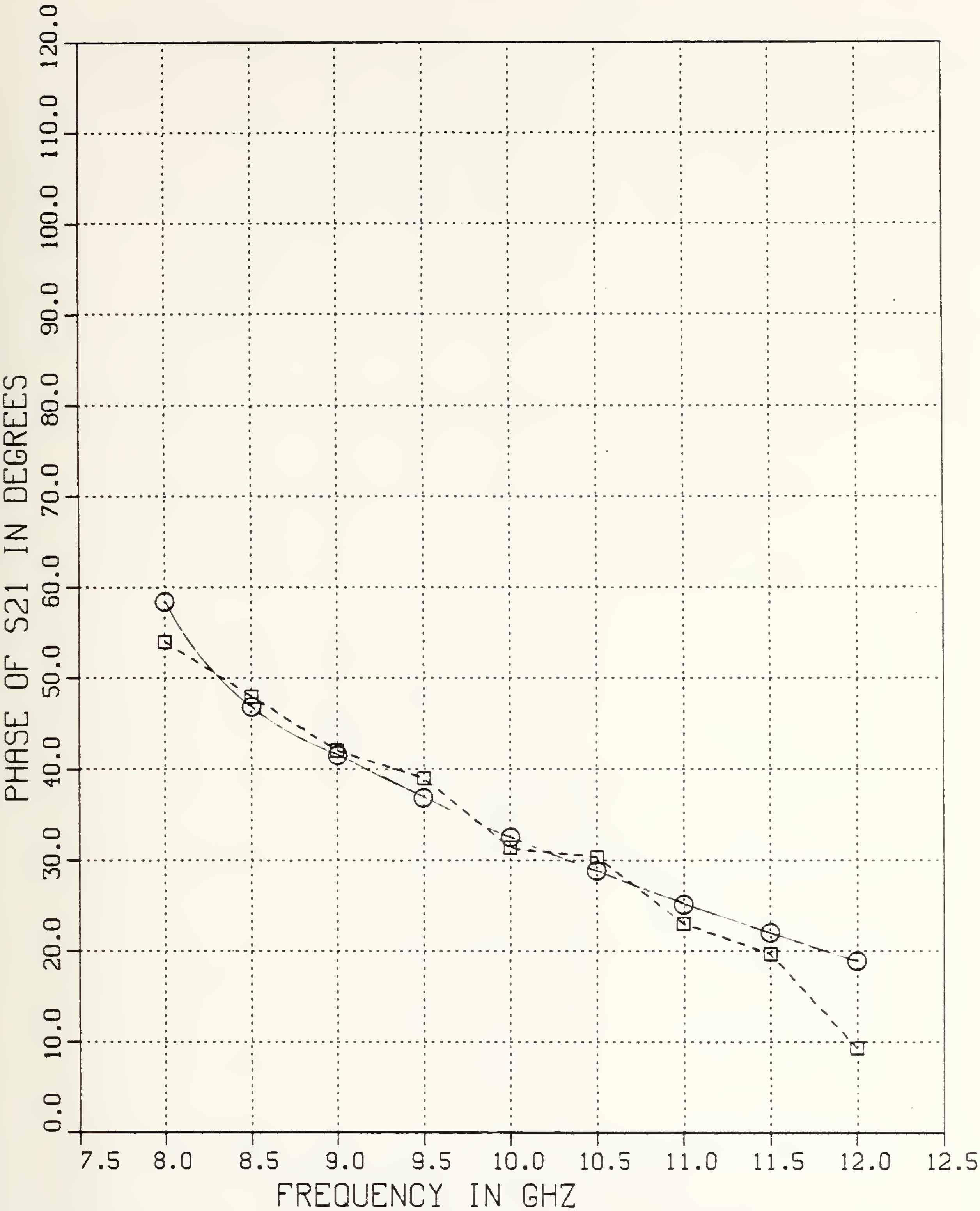


Figure F-20 θ_{21} v.s. Frequency for T=.05 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

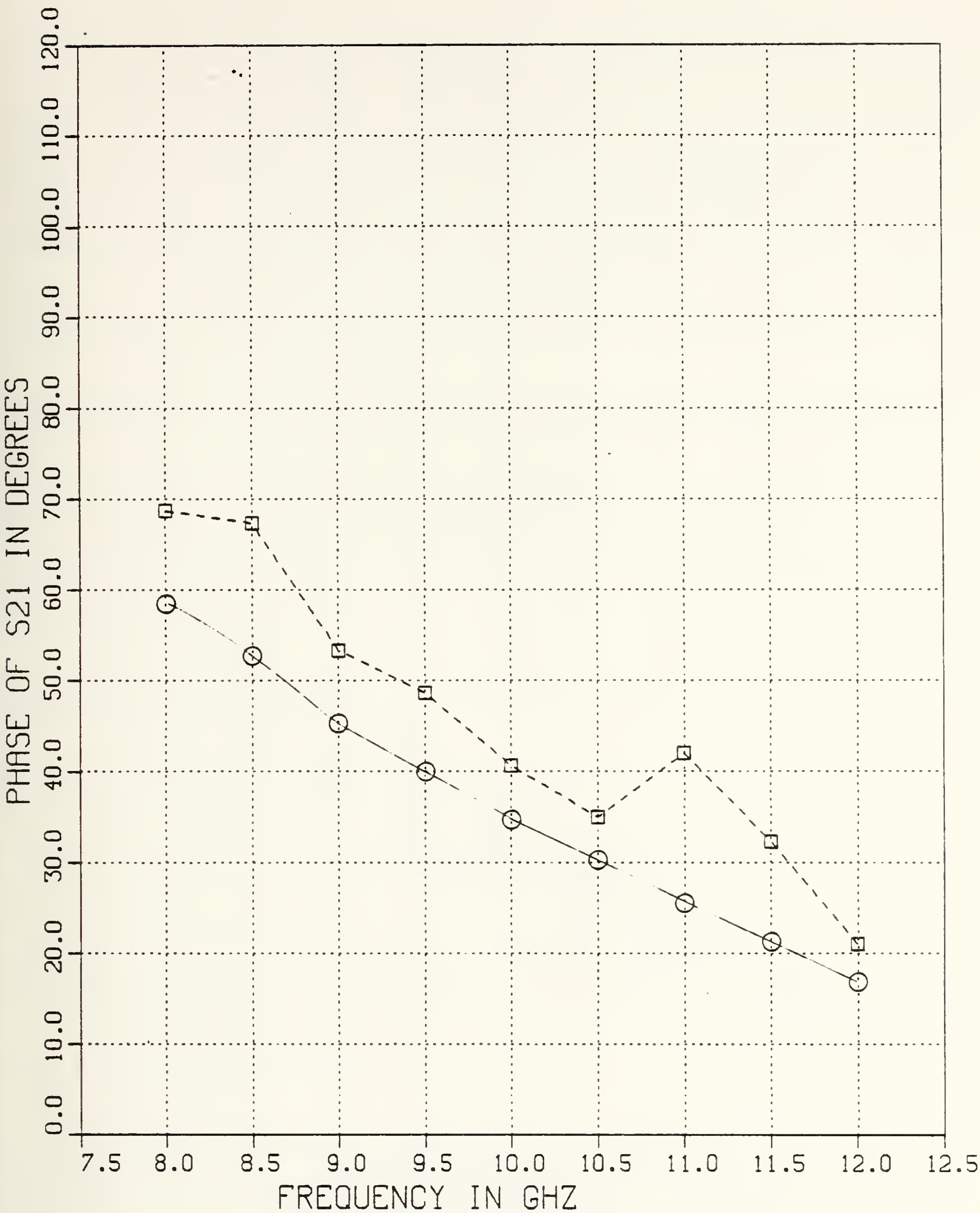


Figure F-21 θ_{21} v.s. Frequency for T=0.1 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

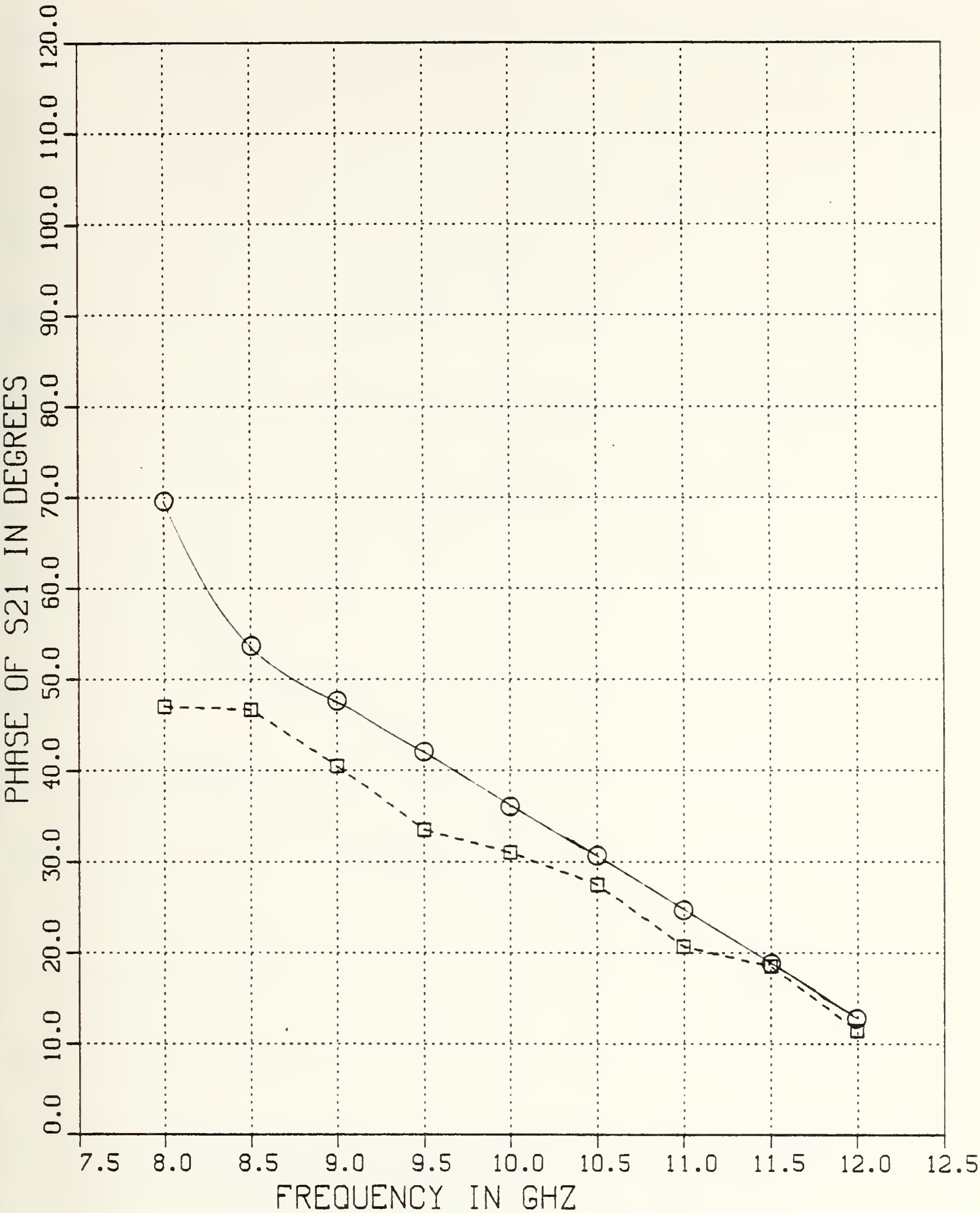


Figure F-22 θ_{21} v.s. Frequency for T=0.2 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

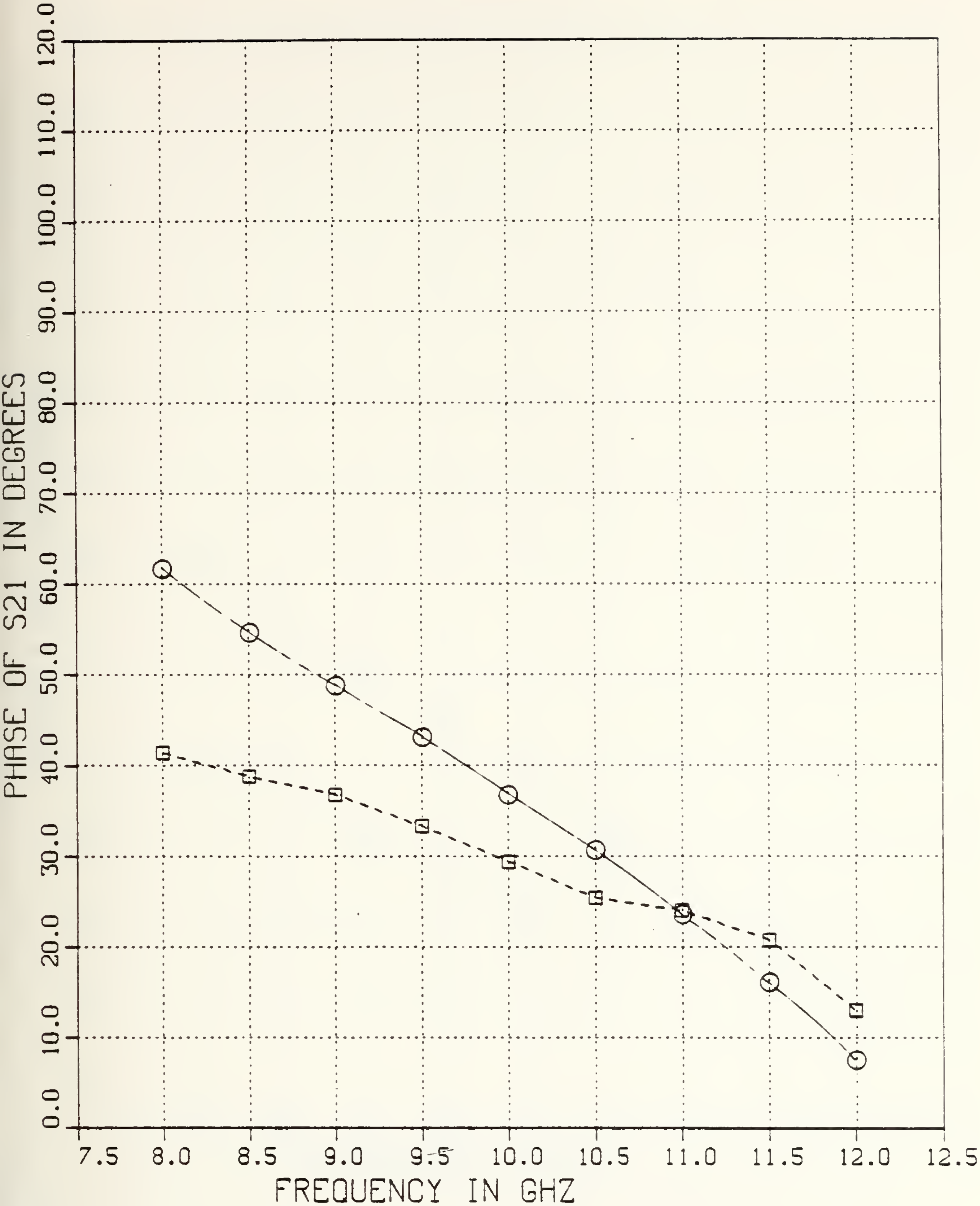


Figure F-23 θ_{21} v.s. Frequency for T=0.5 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

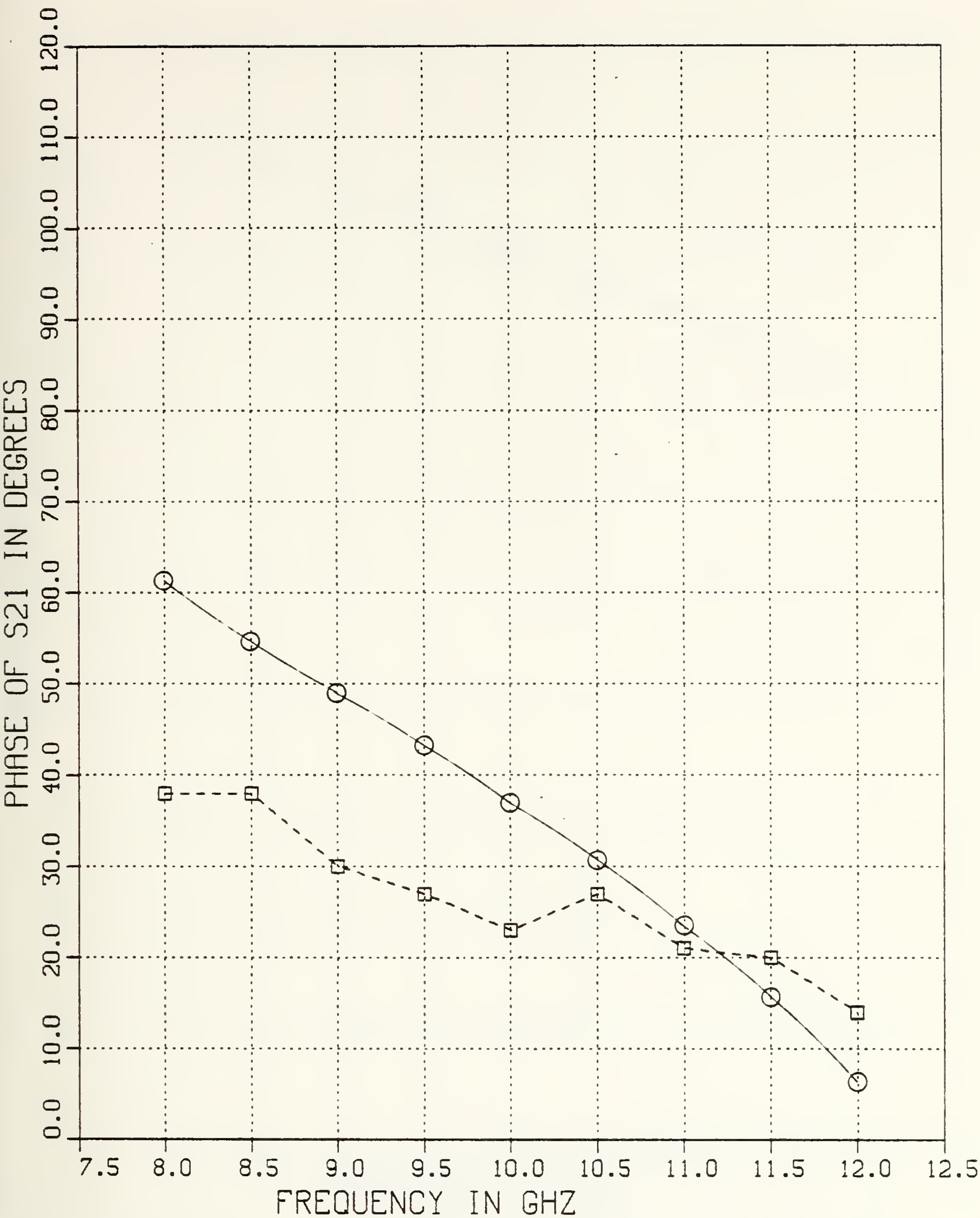


Figure F-24 θ_{21} v.s. Frequency for T=1.0 inch Inductive Strip, w/b=1.0, and ε_{r_2}=1.0

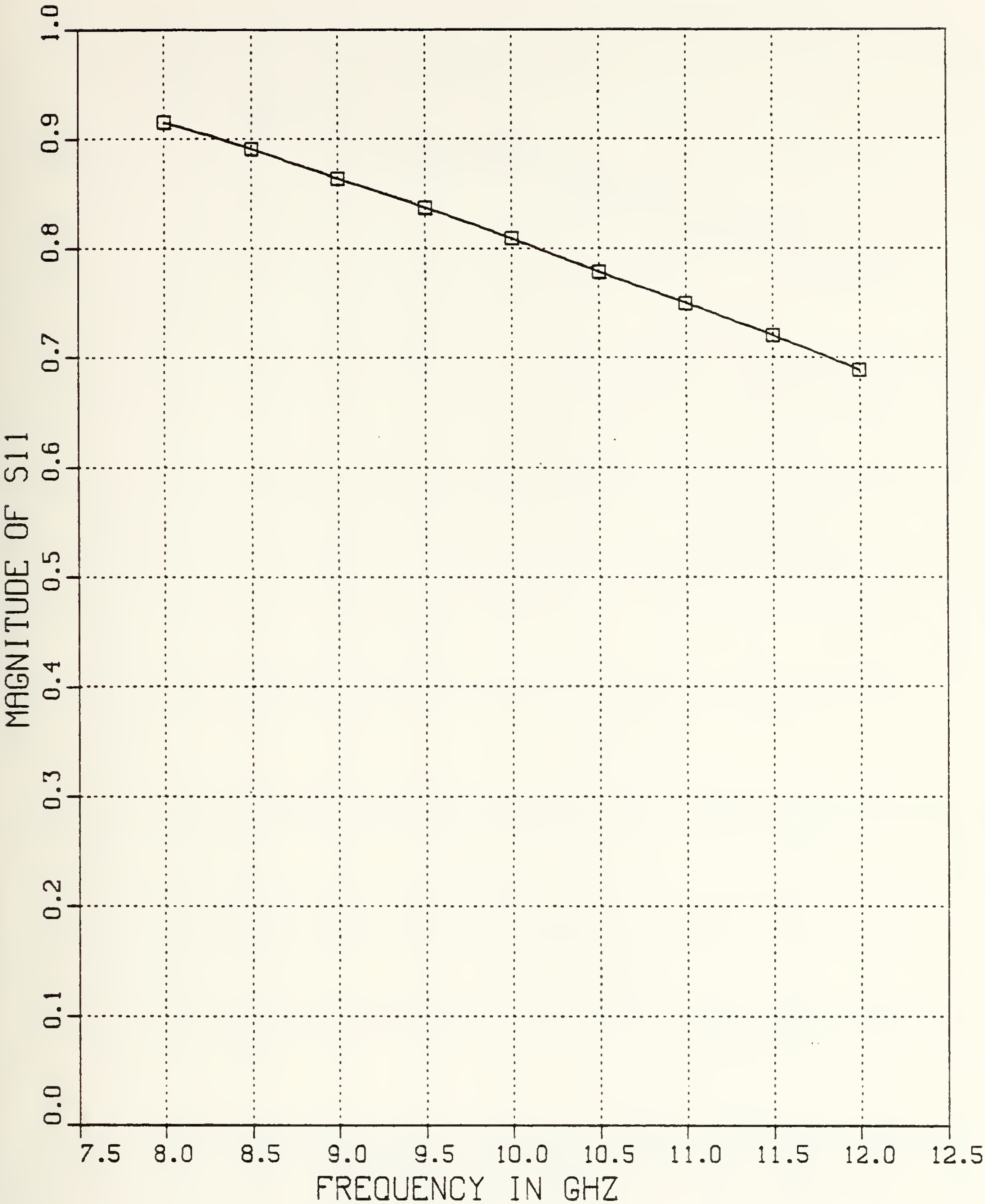


Figure F-25 $|S_{11}|$ v.s. Frequency for T=.05 inch Inductive Strip, w/b=0.5, and ε_{r_2}=1.0

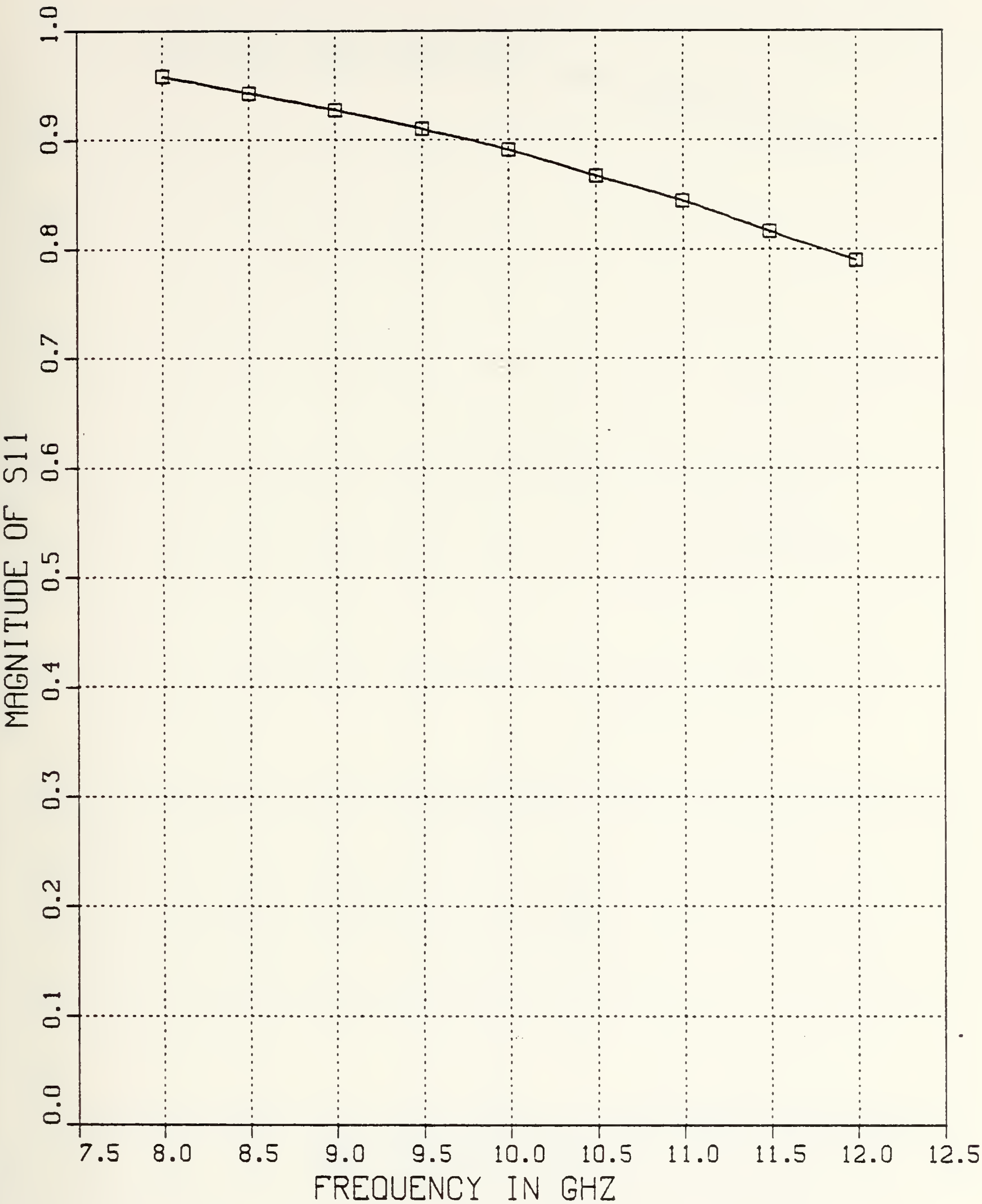


Figure F-26 $|S_{11}|$ v.s. Frequency for T=0.1 inch Inductive Strip, w/b=0.5, and ε_{r_2}=1.0

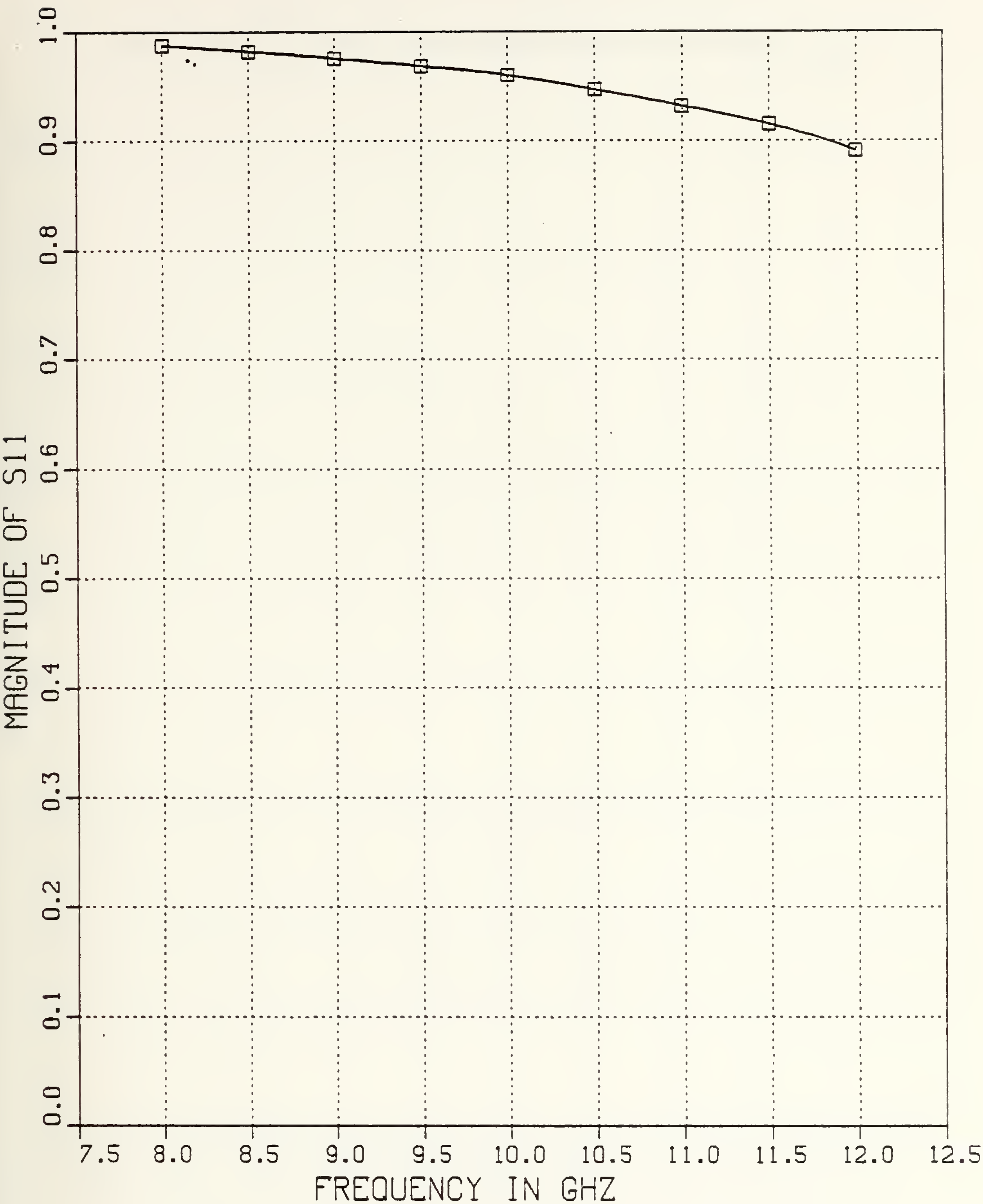


Figure F-27 $|S_{11}|$ v.s. Frequency for T=0.2 inch Inductive Strip, w/b=0.5, and ε_{r_2}=1.0

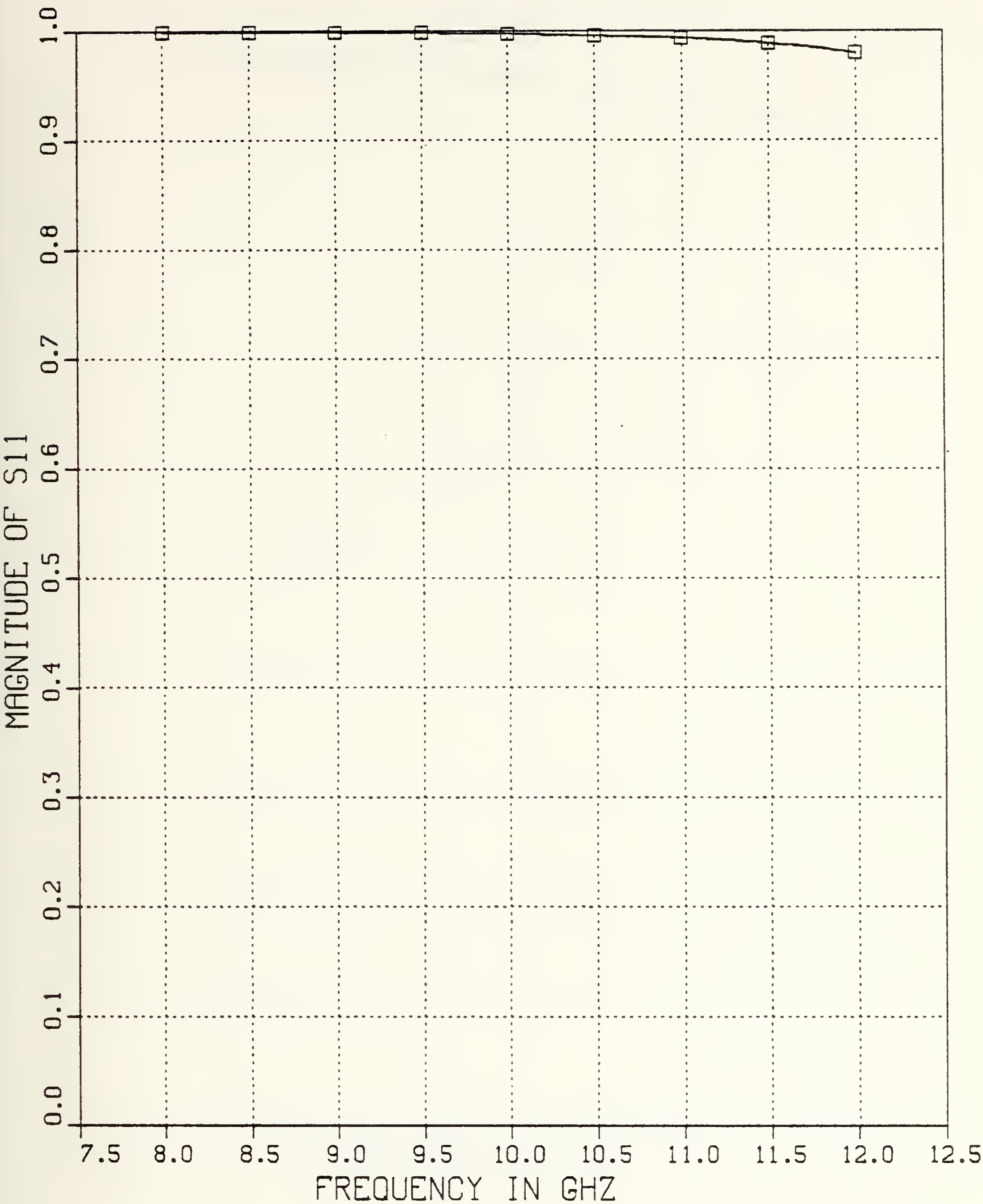


Figure F-28 $|S_{11}|$ v.s. Frequency for T=0.5 inch Inductive Strip, w/b=0.5, and $\varepsilon_{r_2}=1.0$

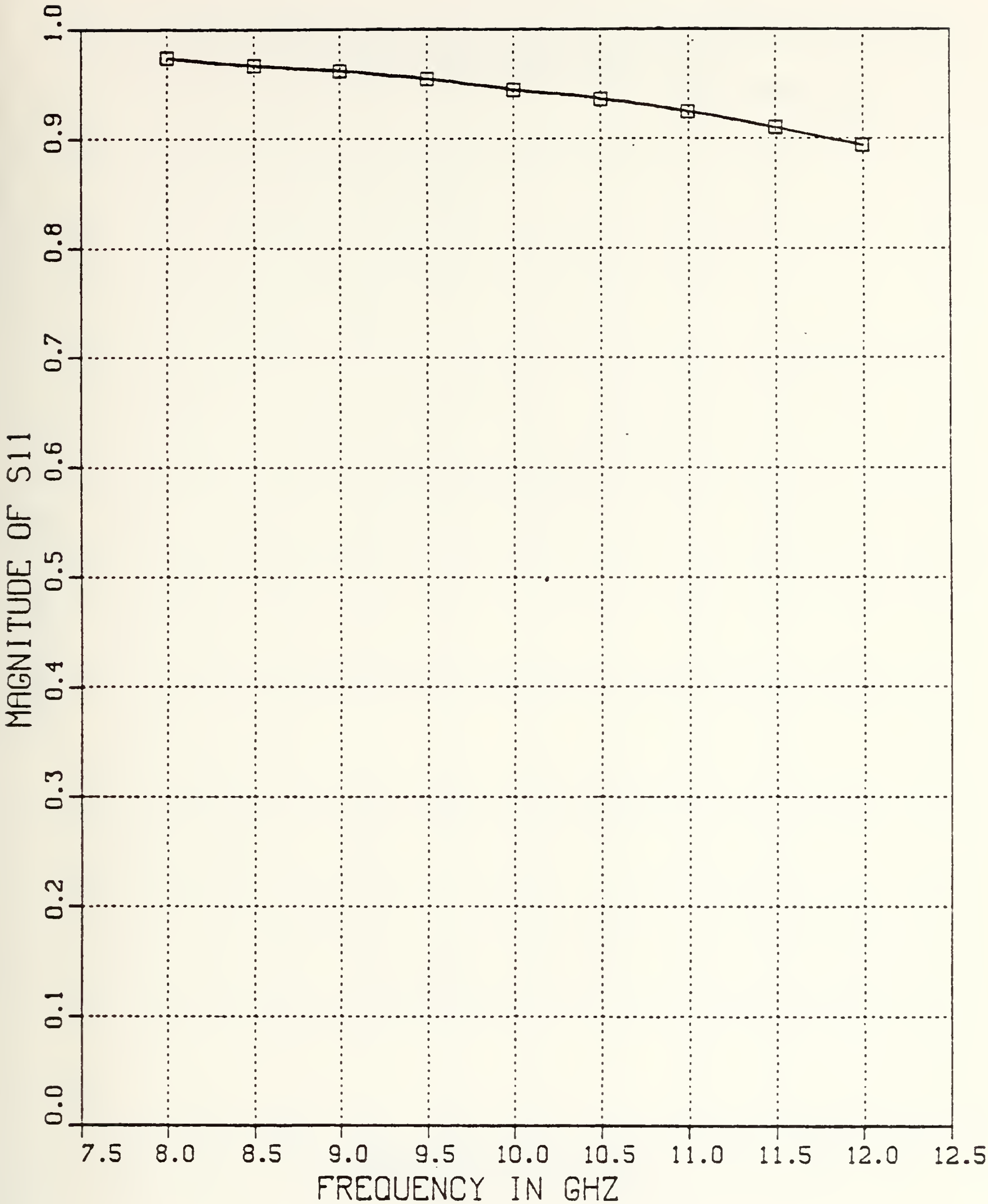


Figure F-29 $|S_{11}|$ v.s. Frequency for T=.05 inch Inductive Strip, w/b=.25, and ε_{r_2}=1.0

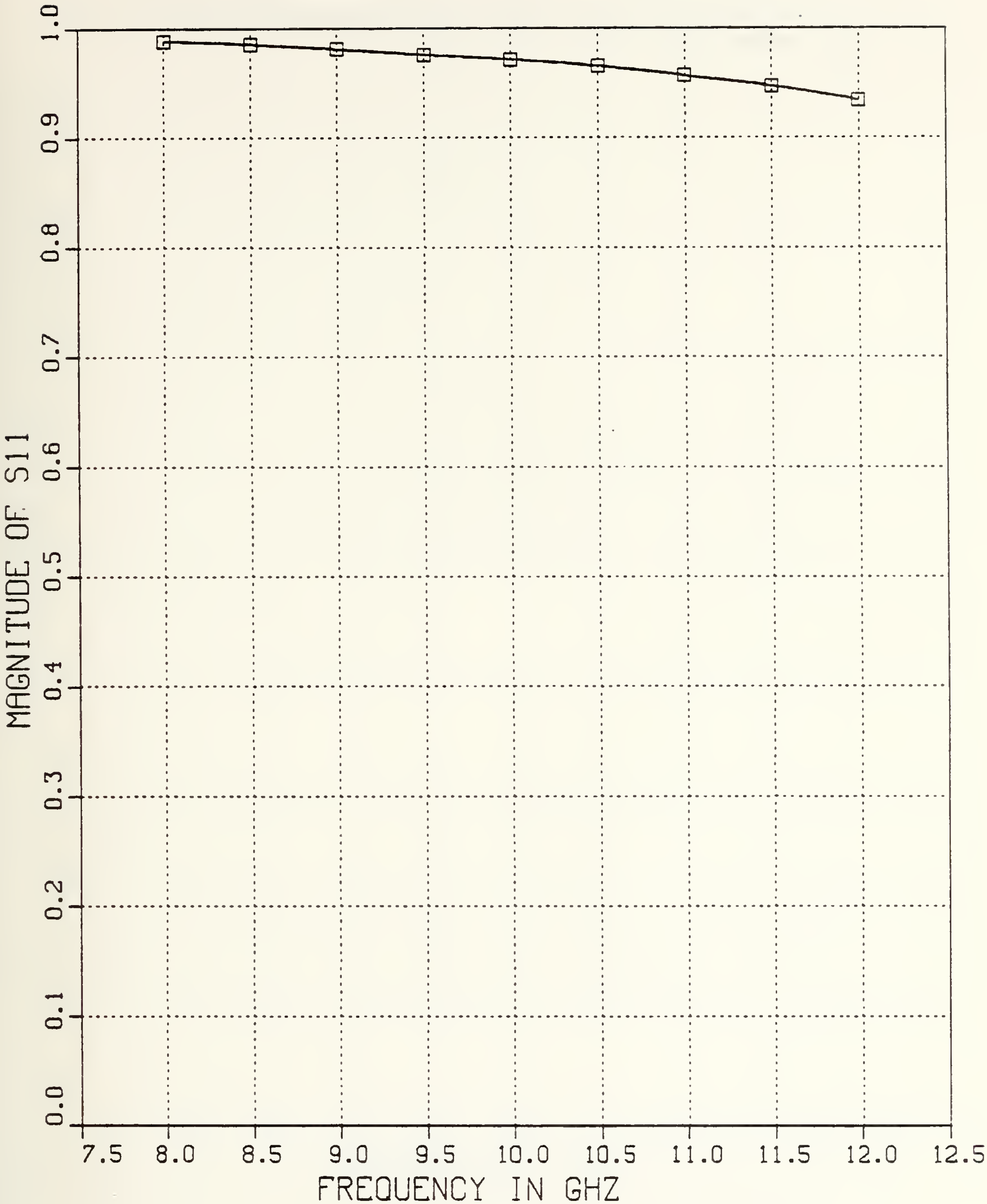


Figure F-30 $|S_{11}|$ v.s. Frequency for T=0.1 inch Inductive Strip, w/b=.25, and ε_{r_2}=1.0

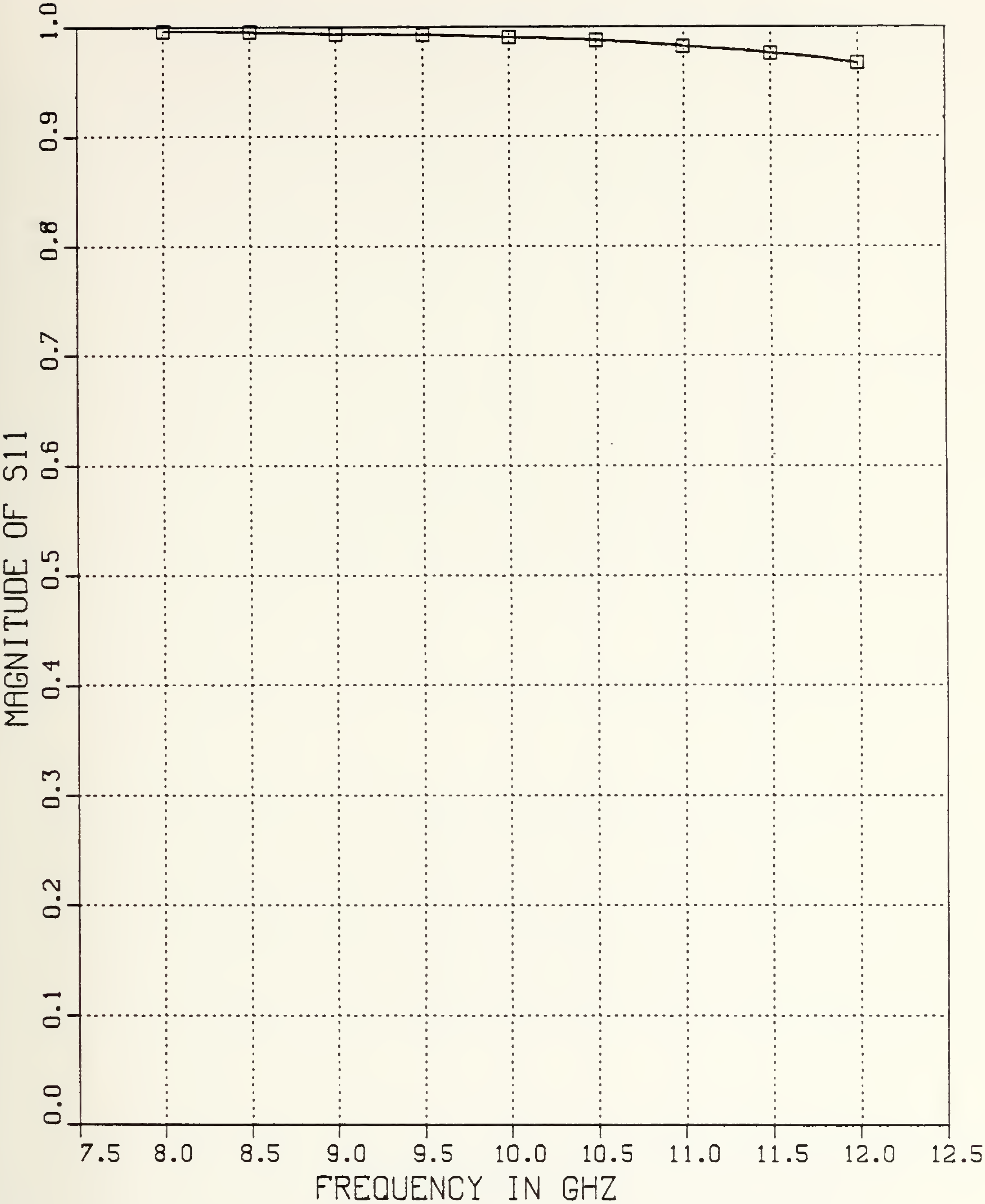


Figure F-31 $|S_{11}|$ v.s. Frequency for T=0.2 inch Inductive Strip, w/b=.25, and ε_{r_2}=1.0

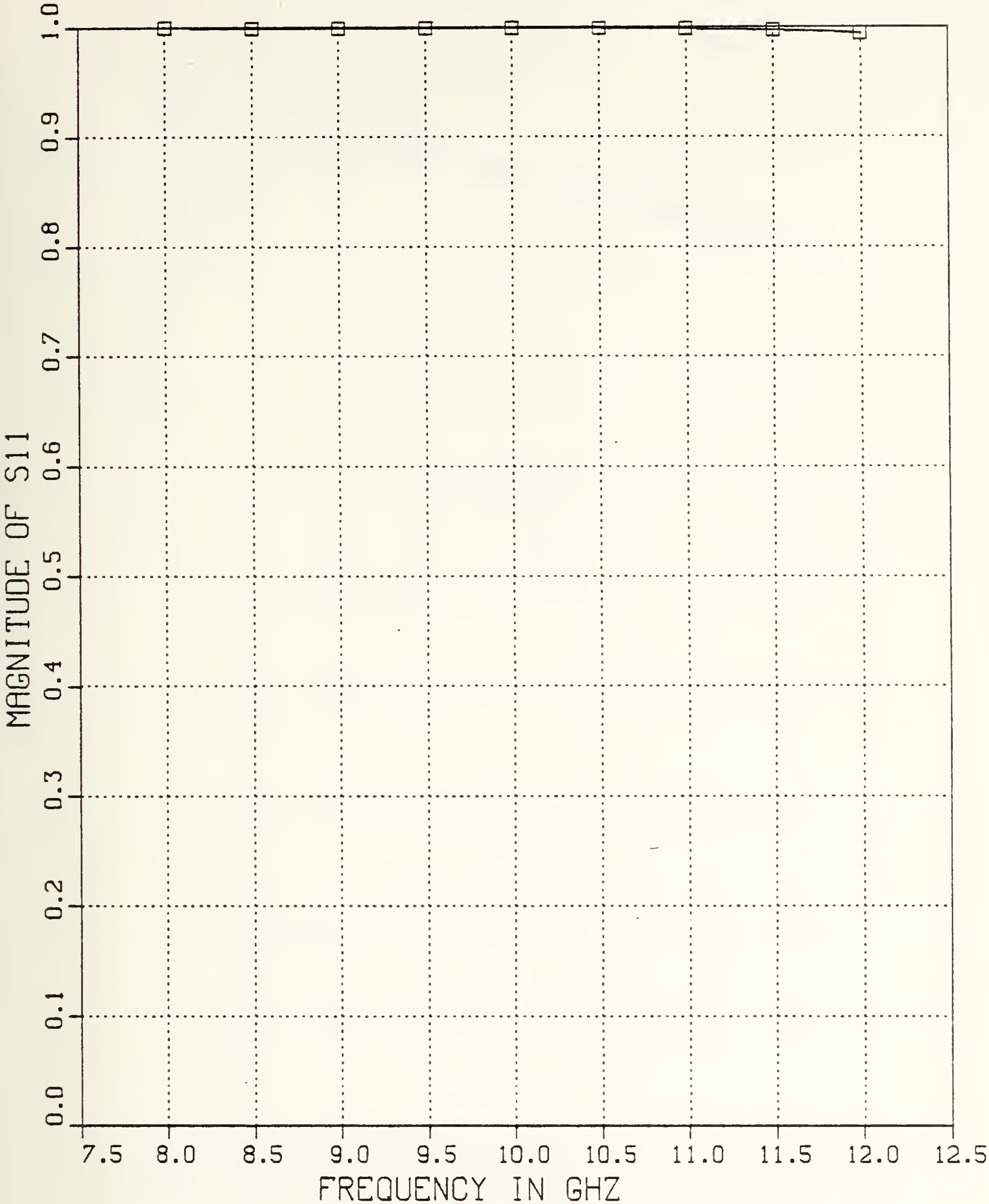


Figure F-32 $|S_{11}|$ v.s. Frequency for T=0.5 inch Inductive Strip, w/b=.25, and ε_{r_2}=1.0

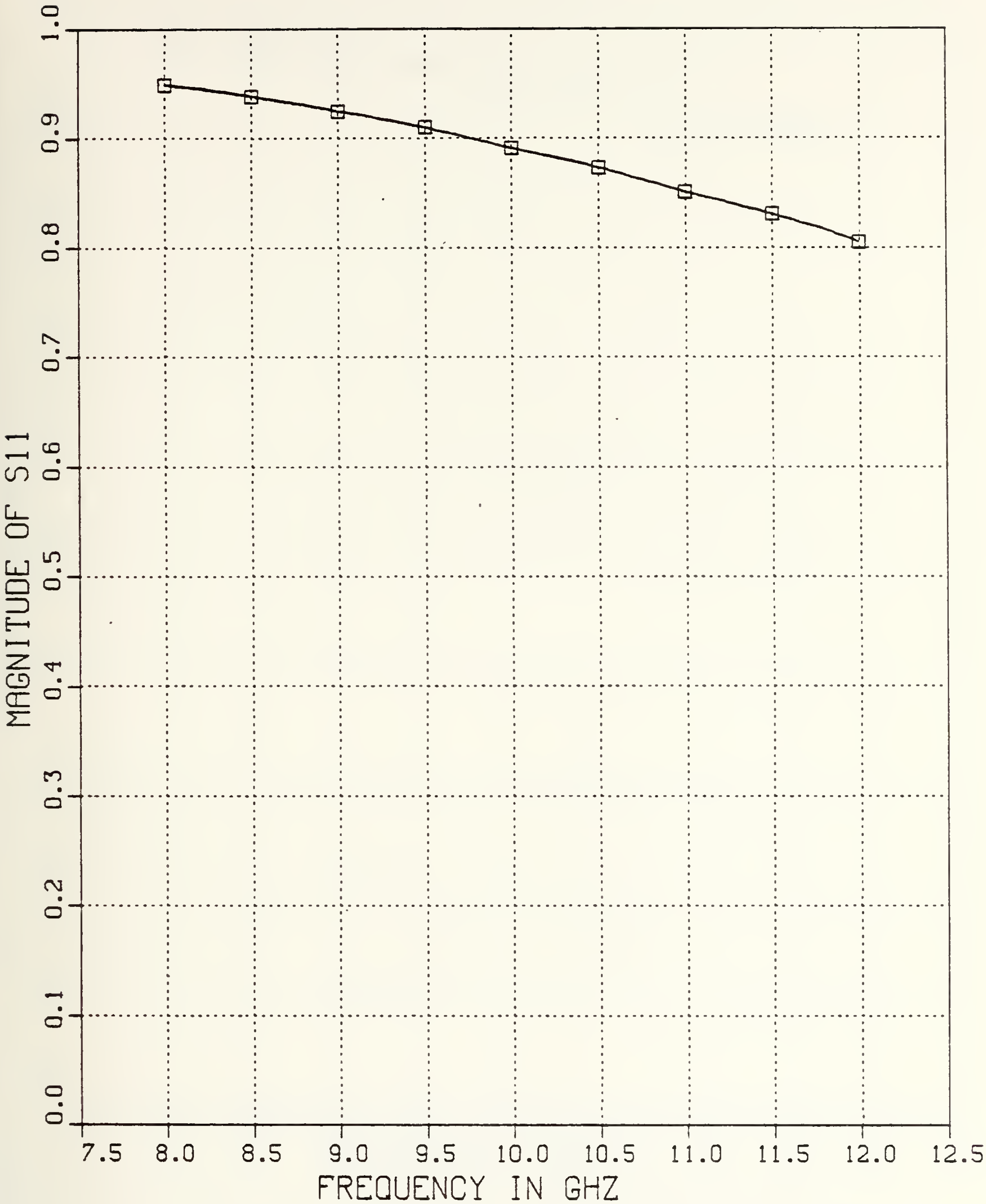


Figure F-33 $|S_{11}|$ v.s. Frequency for T=.05 inch Inductive Strip, w/b=.2, and ε_{r_2}=1.0

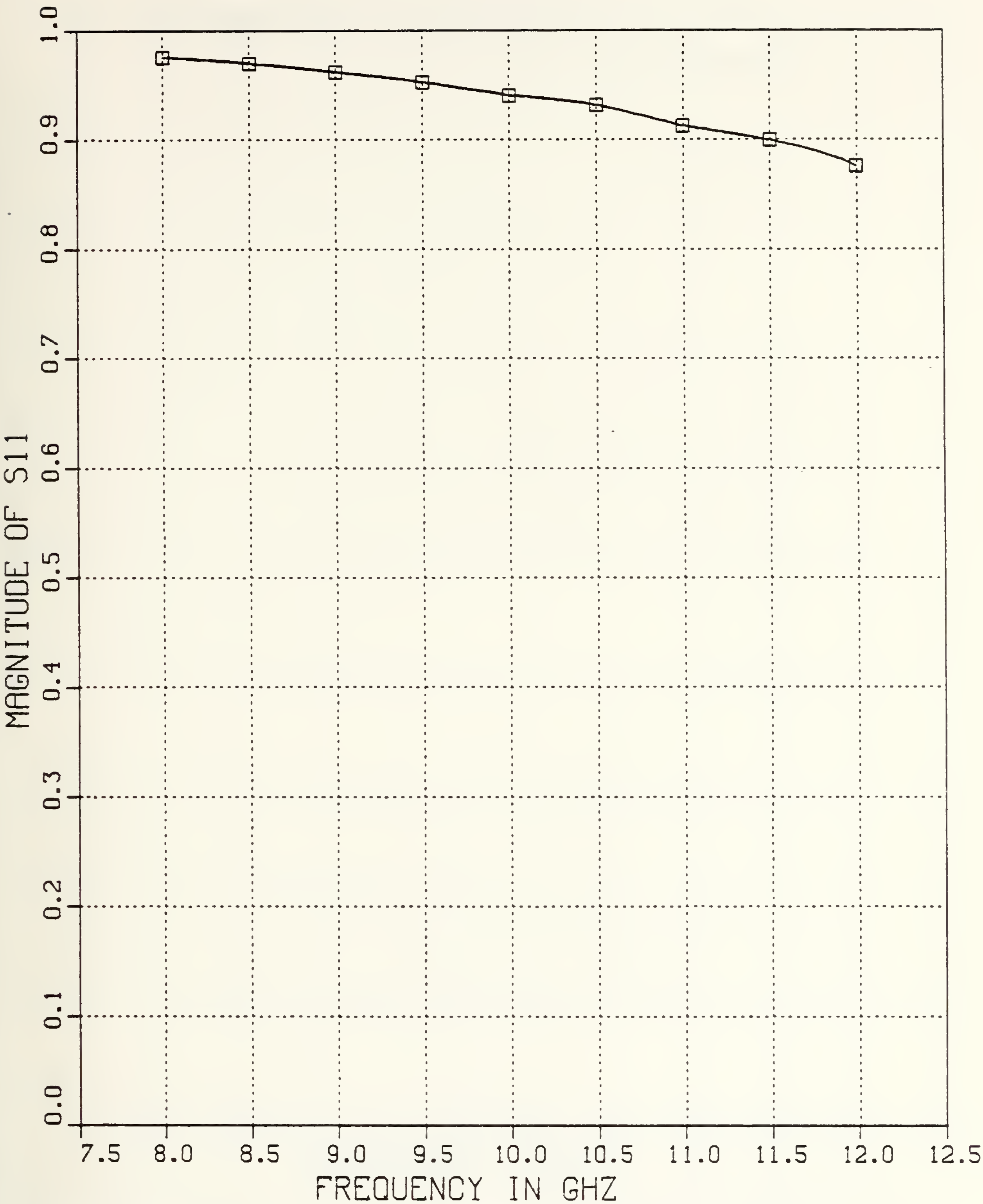


Figure F-34 $|S_{11}|$ v.s. Frequency for T=0.1 inch Inductive Strip, w/b=.2, and ε_{r_2}=1.0

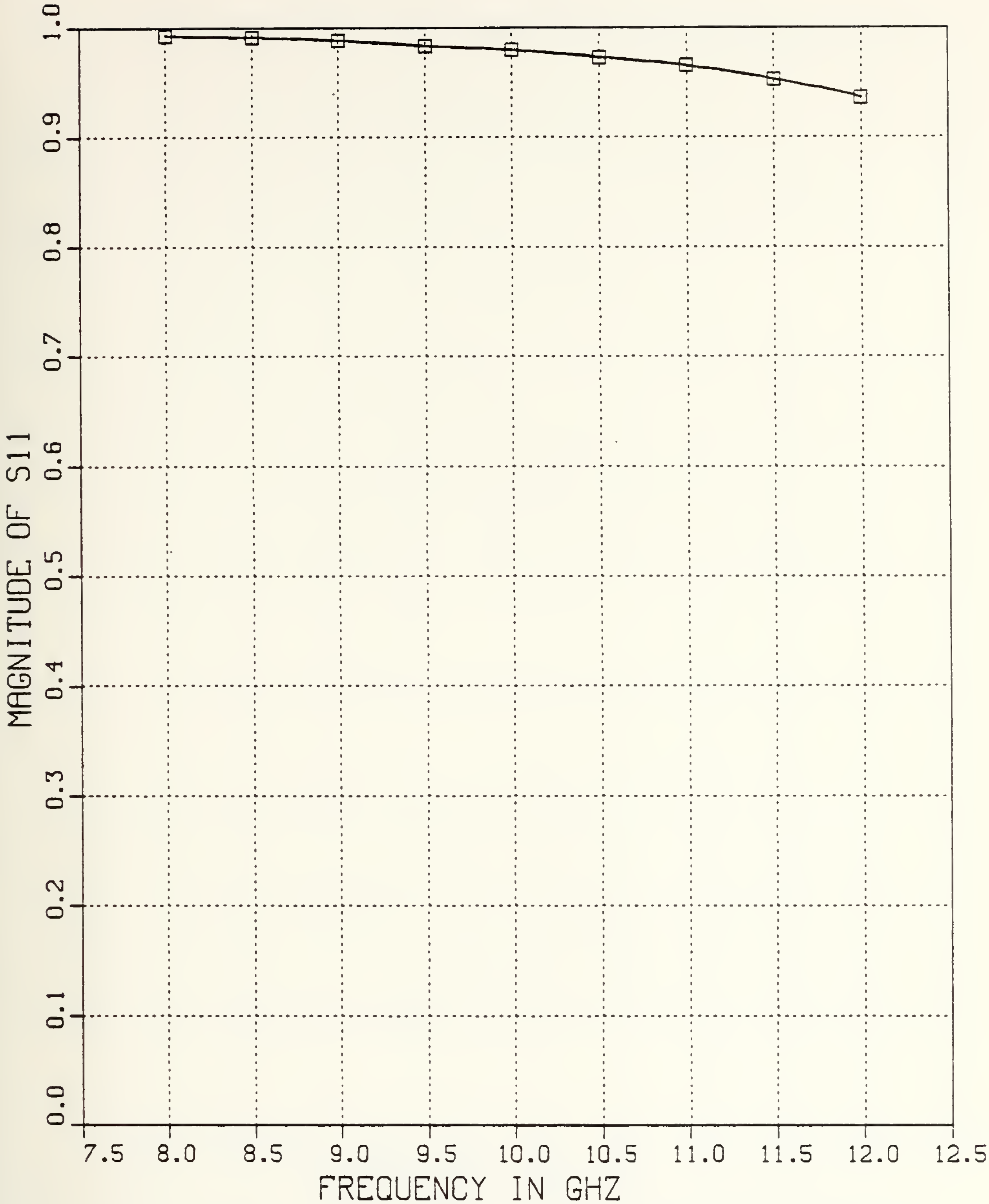


Figure F-35 $|S_{11}|$ v.s. Frequency for T=0.2 inch Inductive Strip, w/b=.2, and ε_{r_2}=1.0

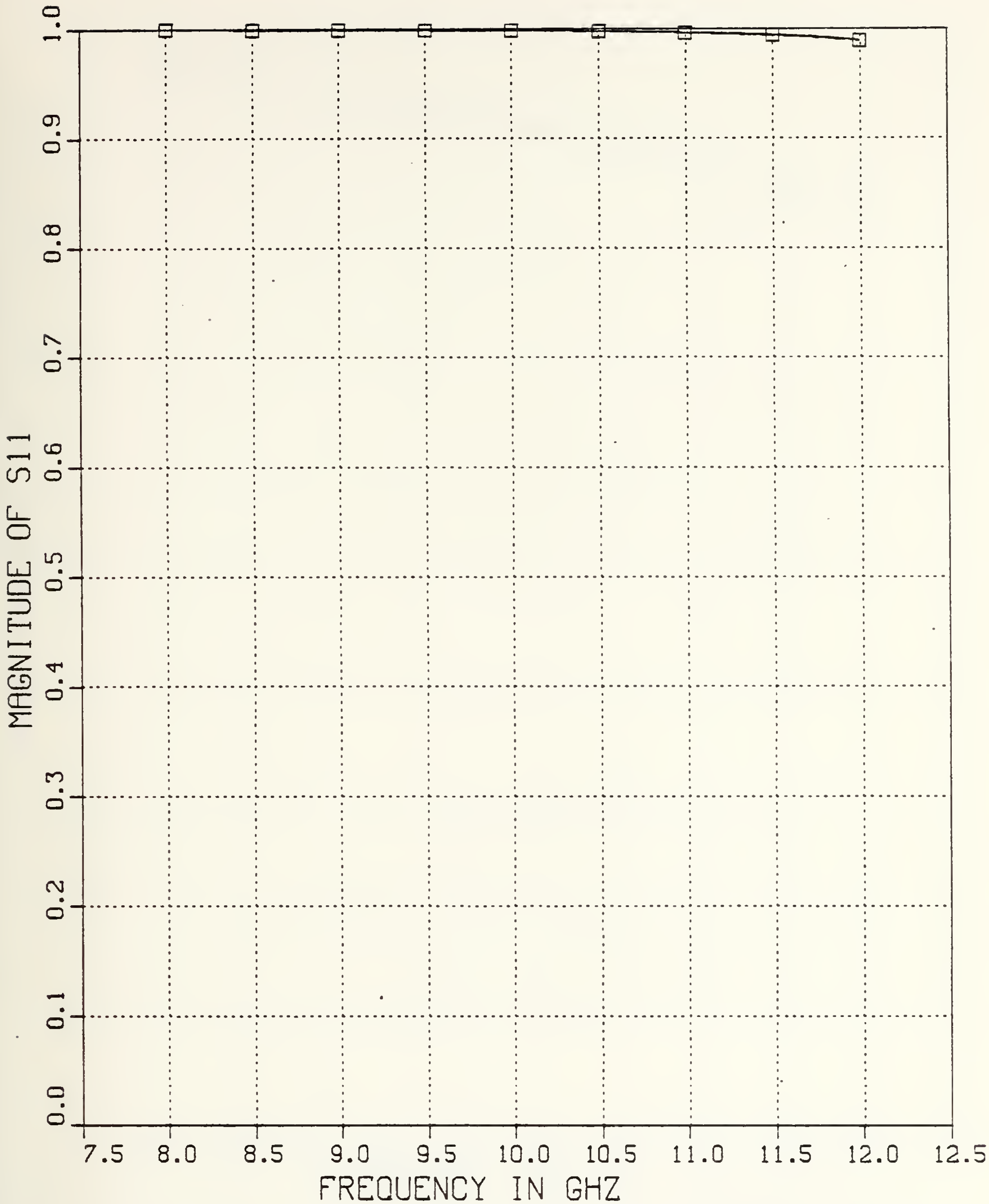


Figure F-36 $|S_{11}|$ v.s. Frequency for T=0.5 inch Inductive Strip, w/b=.2, and ε_{r_2}=1.0

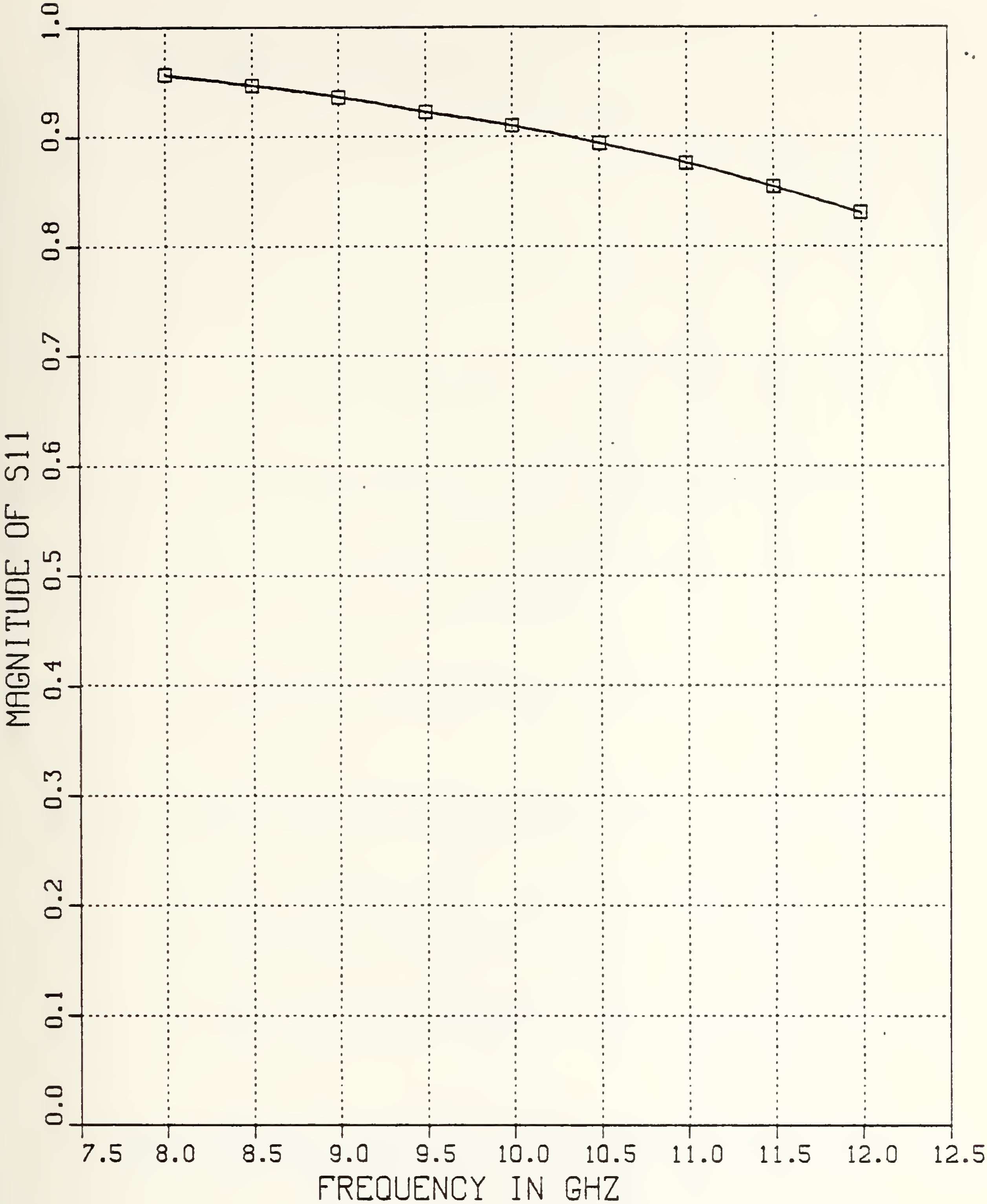


Figure F-37 $|S_{11}|$ v.s. Frequency for T=.05 inch Inductive Strip, w/b=.1, and ε_{r_2}=1.0

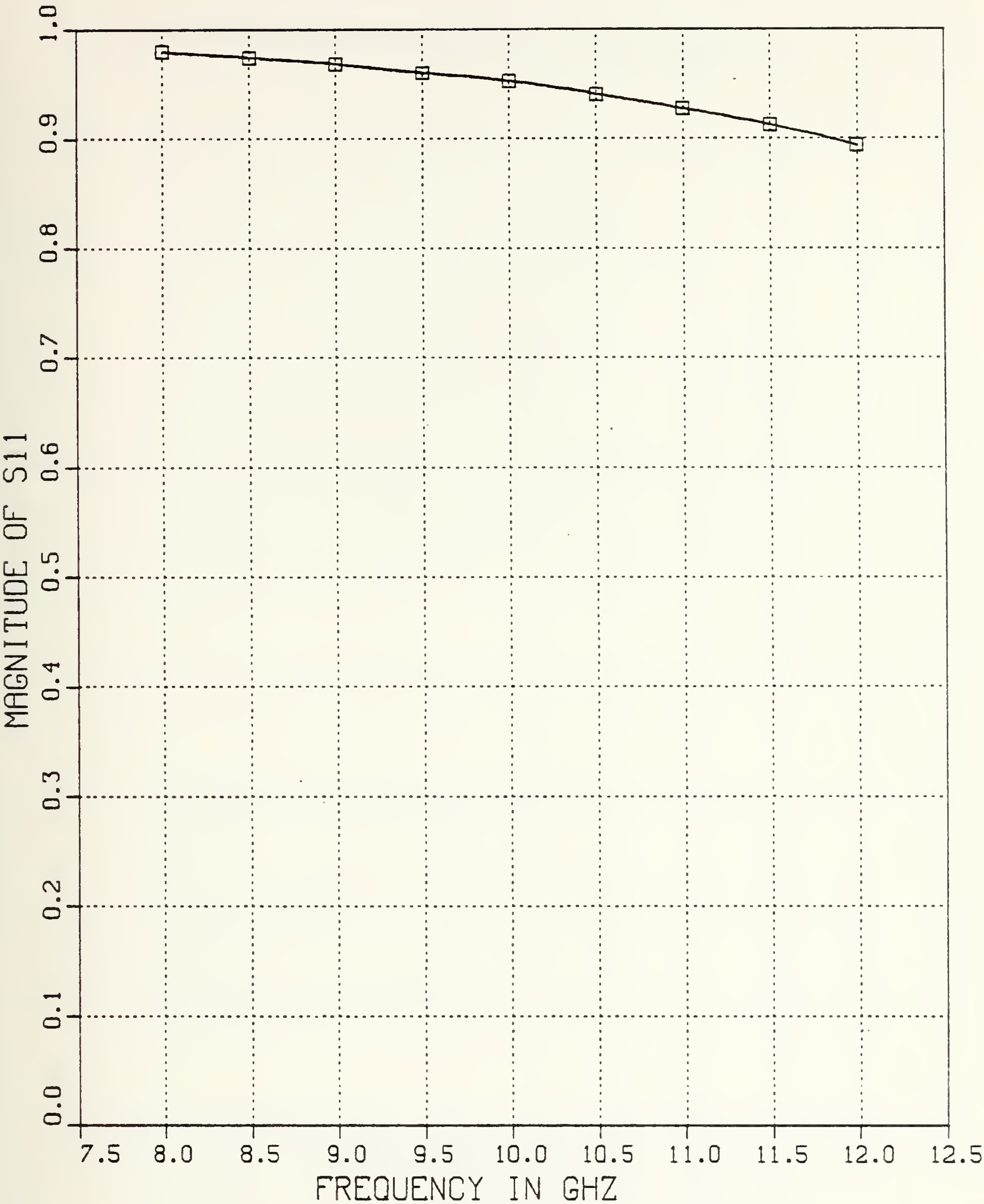


Figure F-38 $|S_{11}|$ v.s. Frequency for T=0.1 inch Inductive Strip, w/b=.1, and ε_{r_2}=1.0

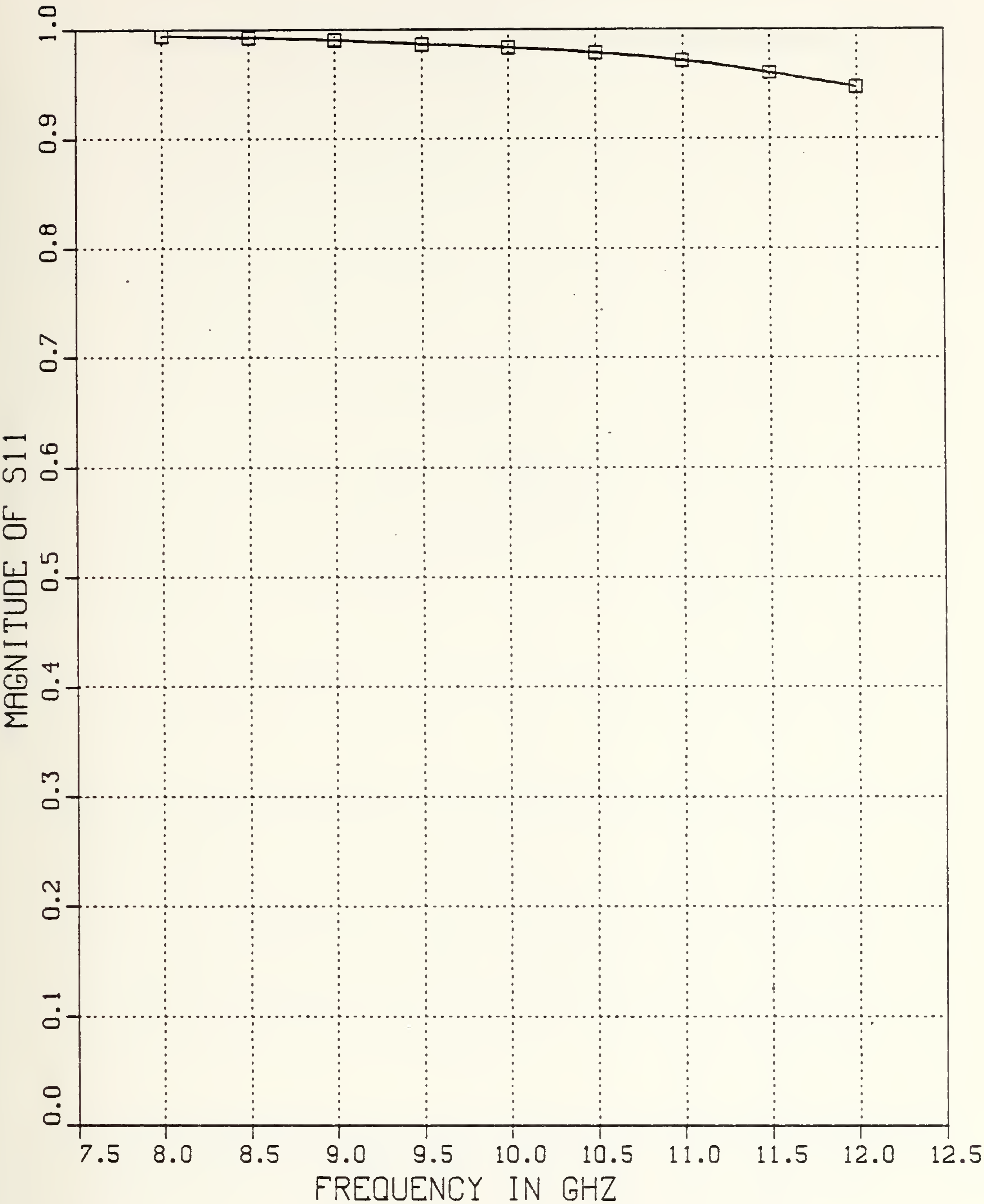


Figure F-39 $|S_{11}|$ v.s. Frequency for T=0.2 inch Inductive Strip, w/b=.1, and ε_{r_2}=1.0

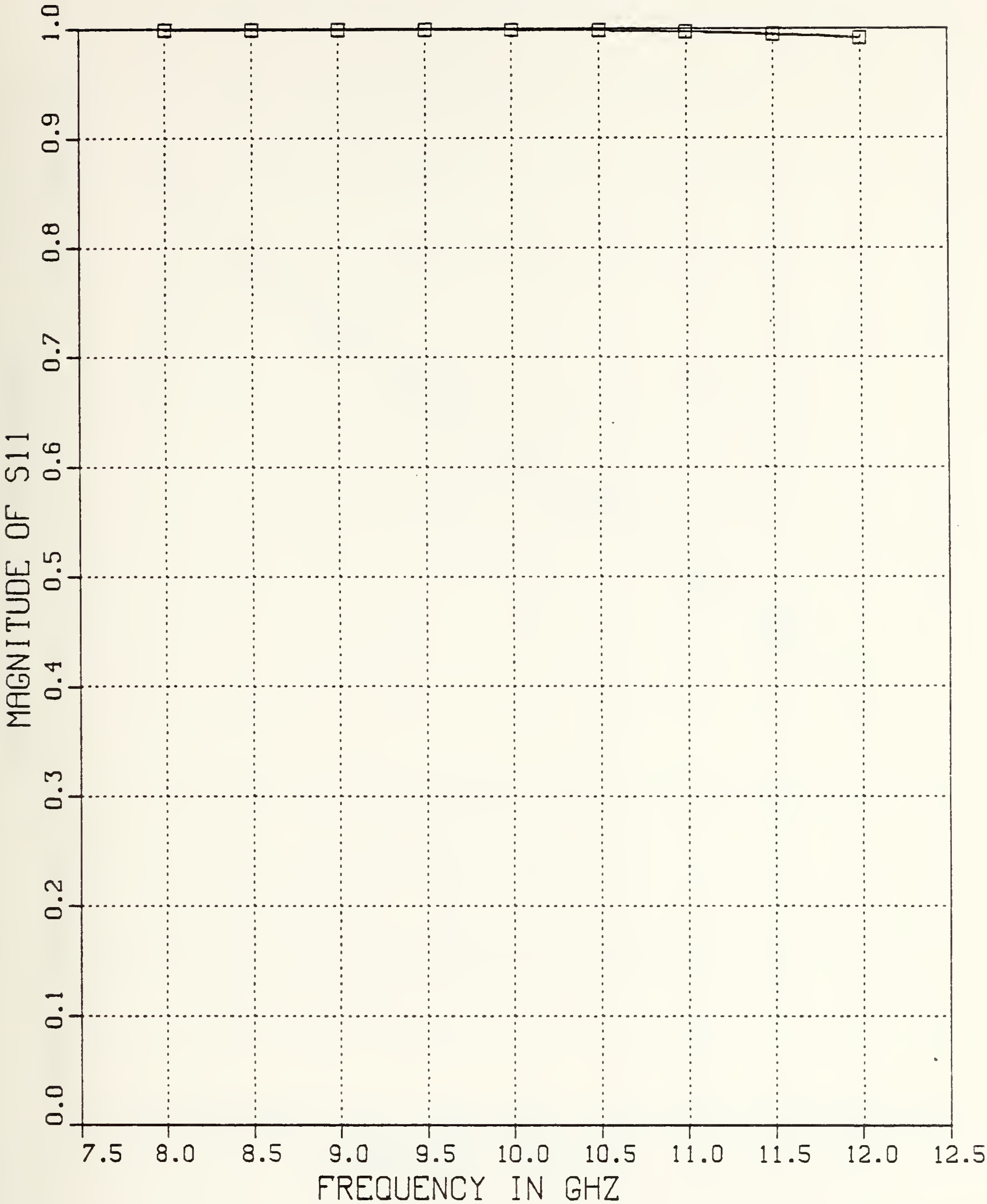


Figure F-40 $|S_{11}|$ v.s. Frequency for T=0.5 inch Inductive Strip, w/b=.1, and ε_{r_2}=1.0

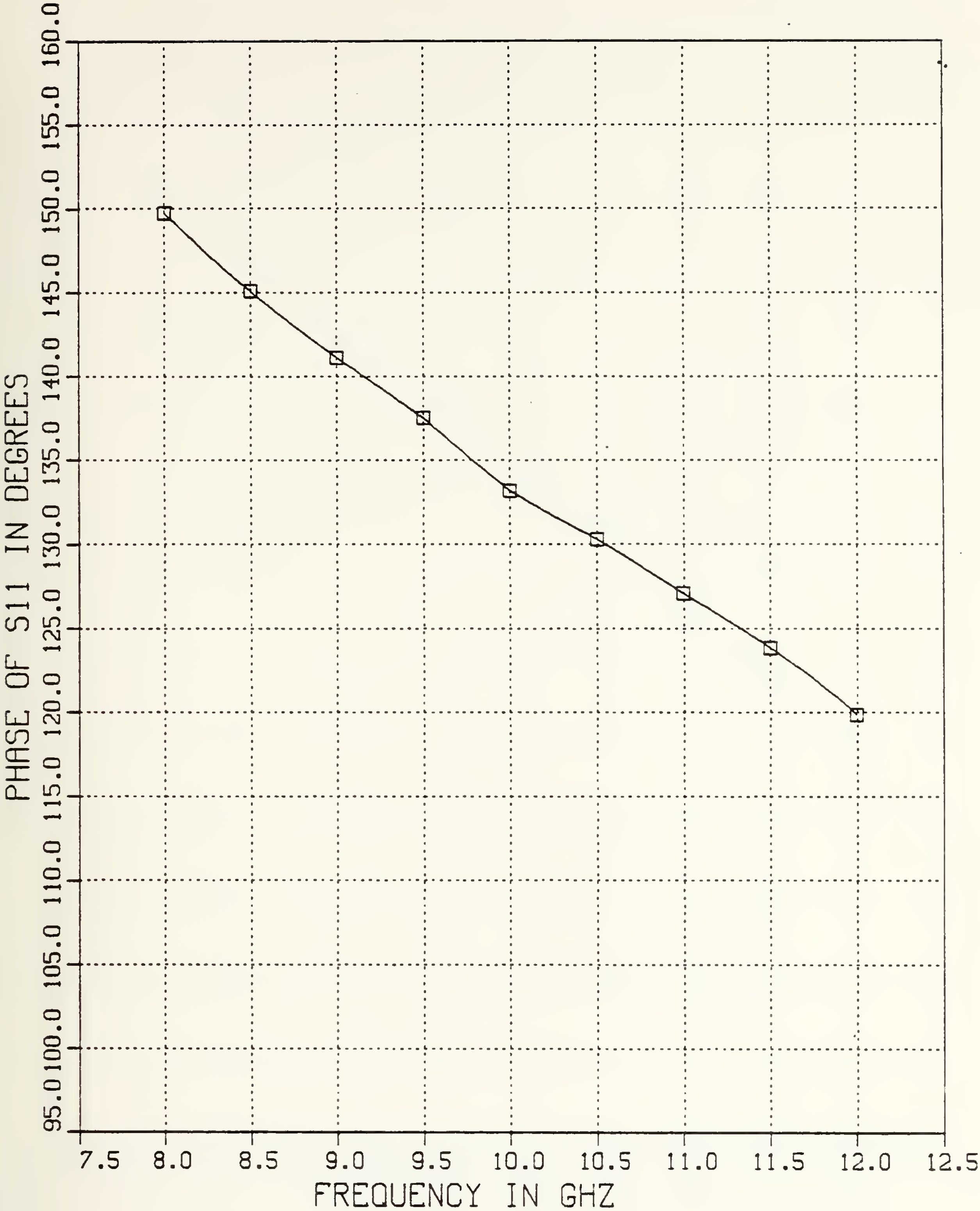


Figure F-41 θ_{11} v.s. Frequency for T=.05 inch Inductive Strip, w/b=.5, and ε_{r_2}=1.0

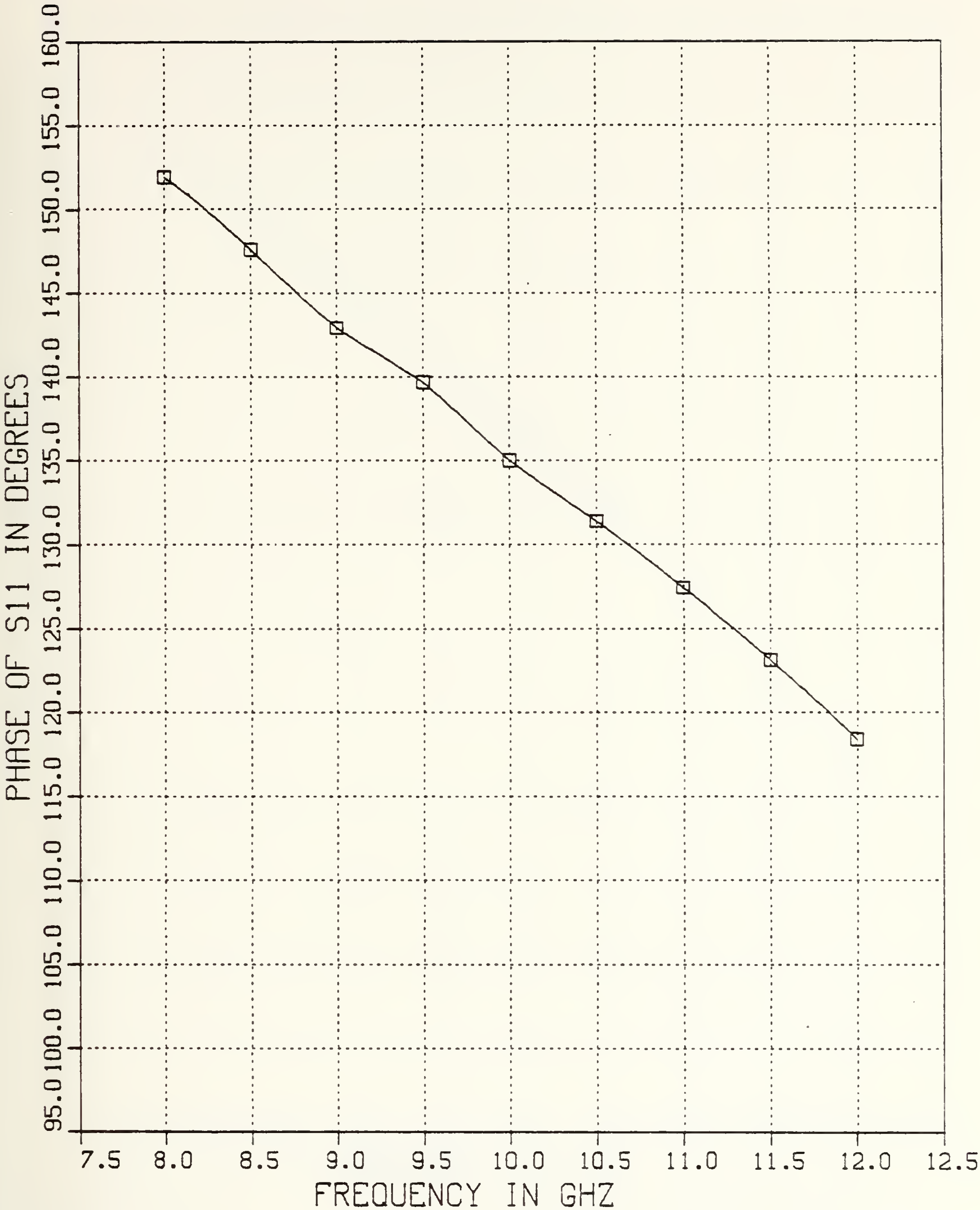


Figure F-42 θ_{11} v.s. Frequency for T=0.1 inch Inductive Strip, w/b=.5, and ε_{r_2}=1.0

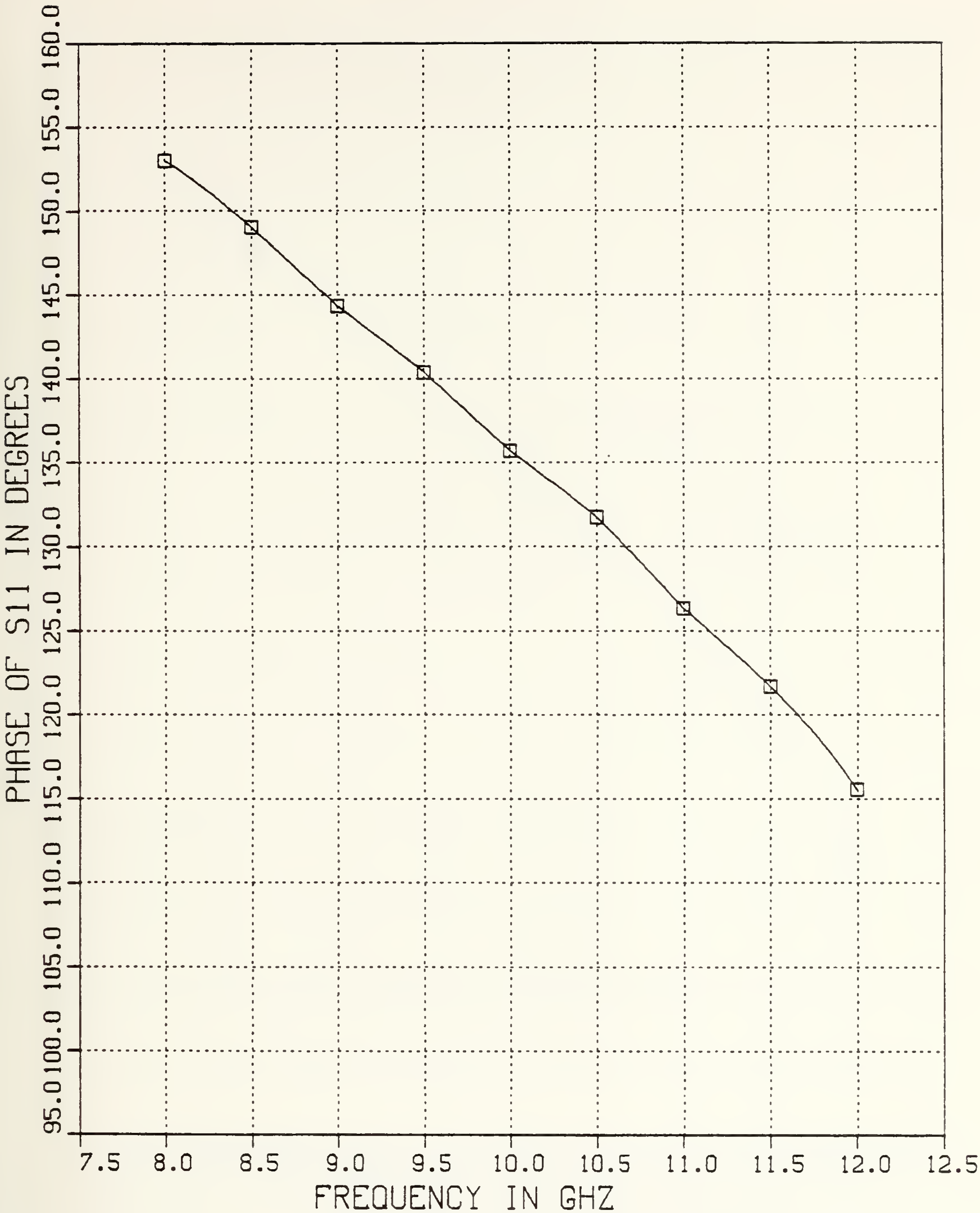


Figure F-43 θ_{11} v.s. Frequency for T=0.2 inch Inductive Strip, w/b=.5, and ε_{r_2}=1.0

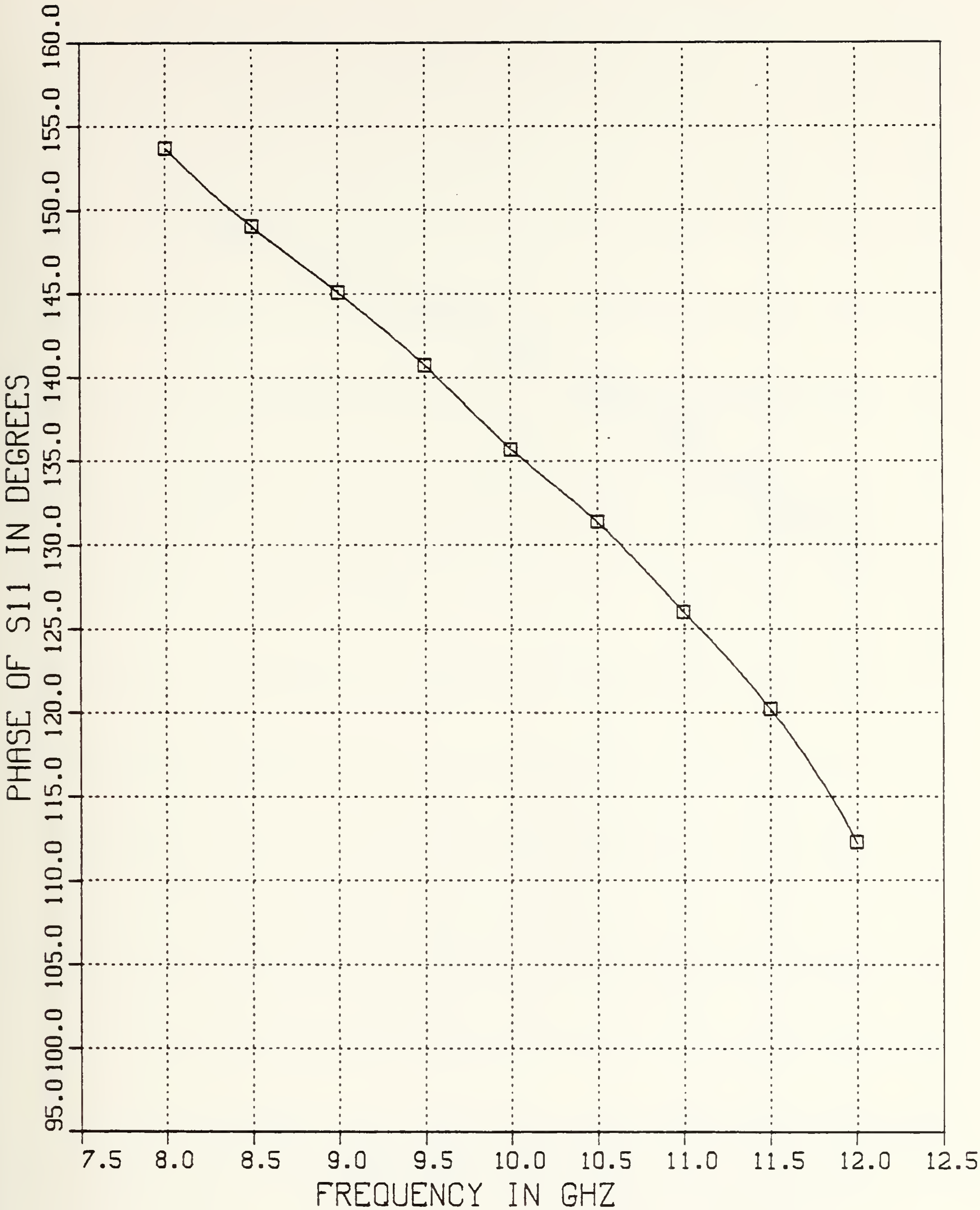


Figure F-44 θ_{11} v.s. Frequency for T=0.5 inch Inductive Strip, w/b=.5, and ε_{r_2}=1.0

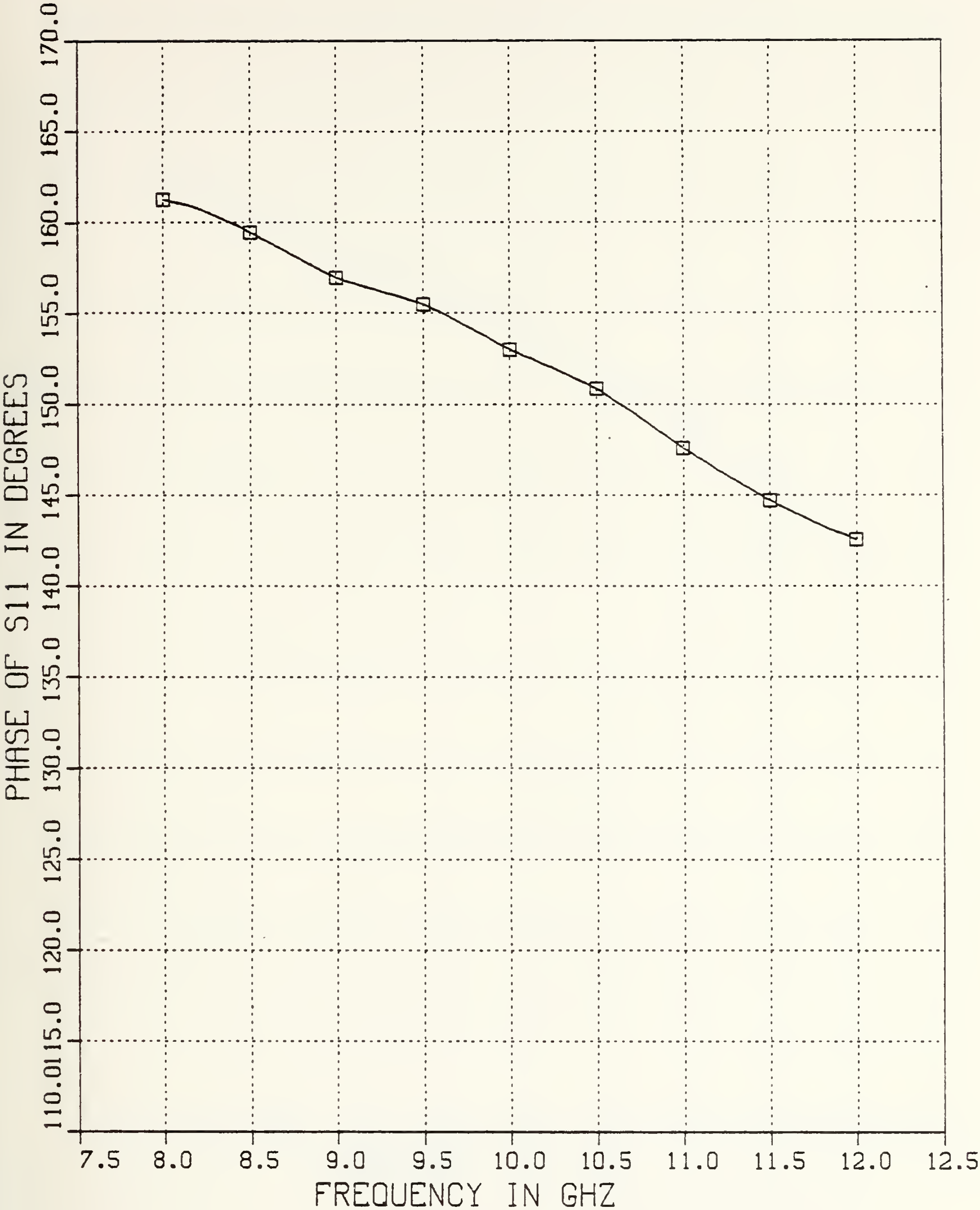


Figure F-45 θ_{11} v.s. Frequency for T=.05 inch Inductive Strip, w/b=.25, and ε_{r_2}=1.0

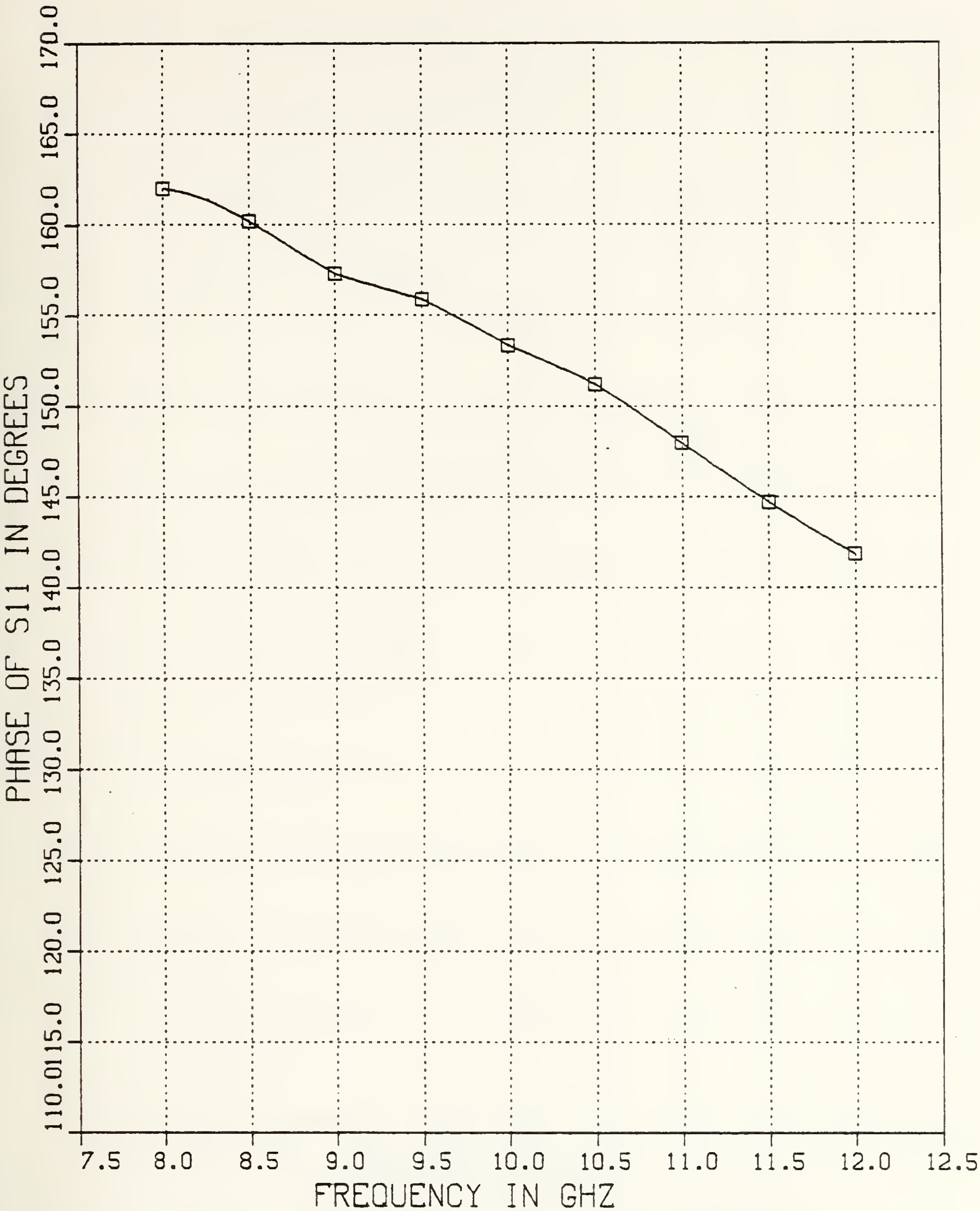


Figure F-46 θ_{11} v.s. Frequency for T=0.1 inch Inductive Strip, w/b=.25, and ε_{r_2}=1.0

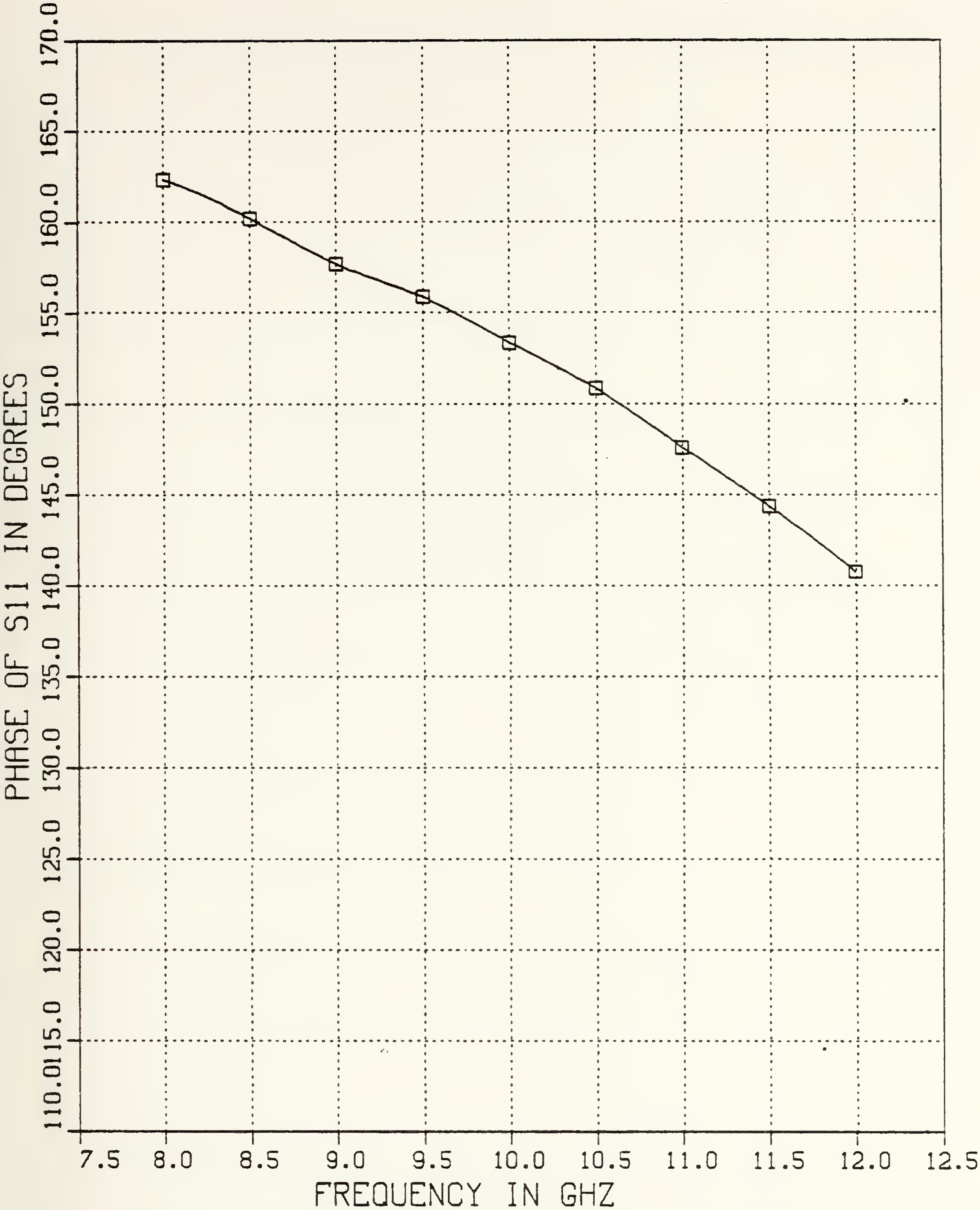


Figure F-47 θ_{11} v.s. Frequency for T=0.2 inch Inductive Strip, w/b=.25, and ε_{r_2}=1.0

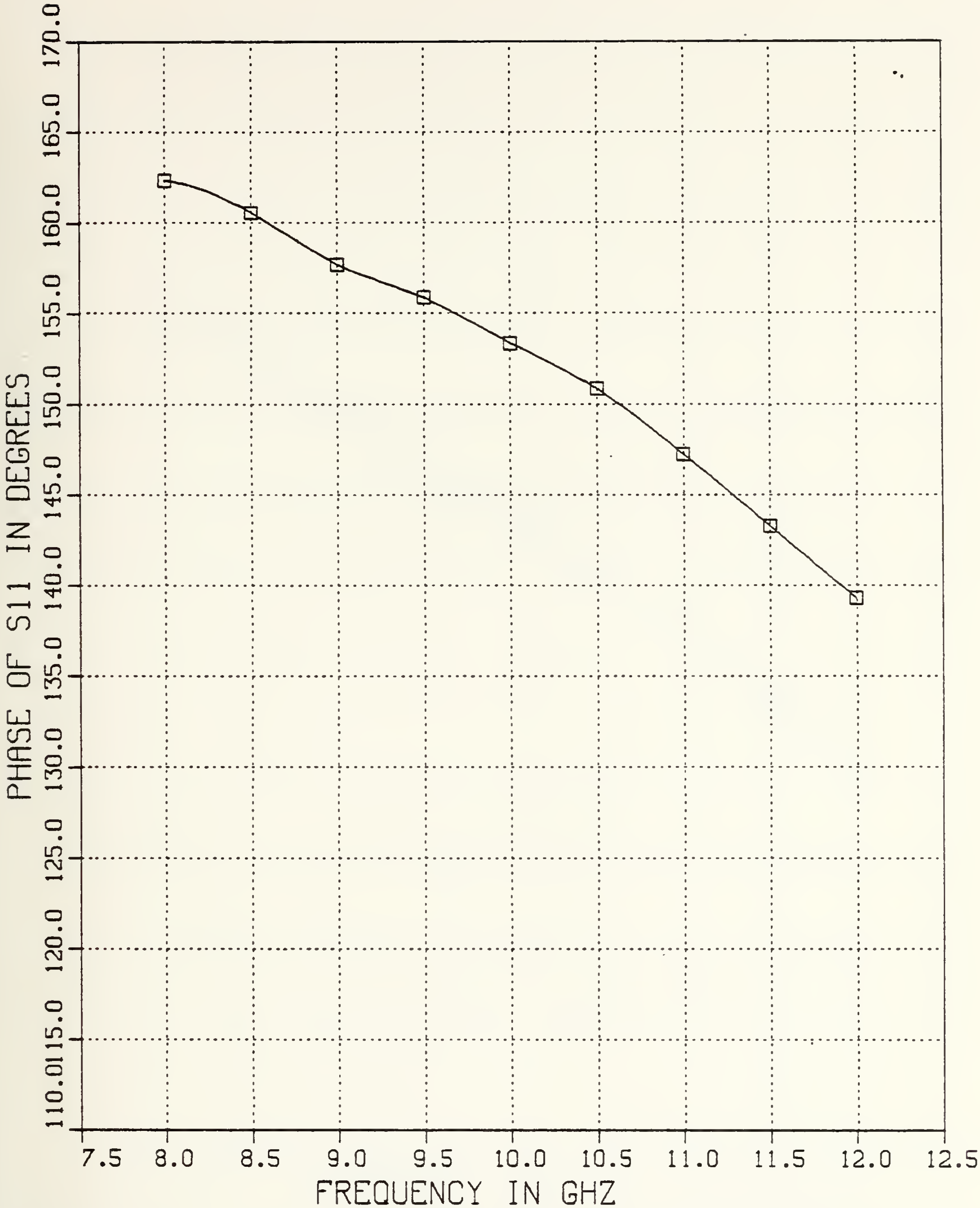


Figure F-48 θ_{11} v.s. Frequency for T=0.5 inch Inductive Strip, w/b=.25, and ε_{r_2}=1.0

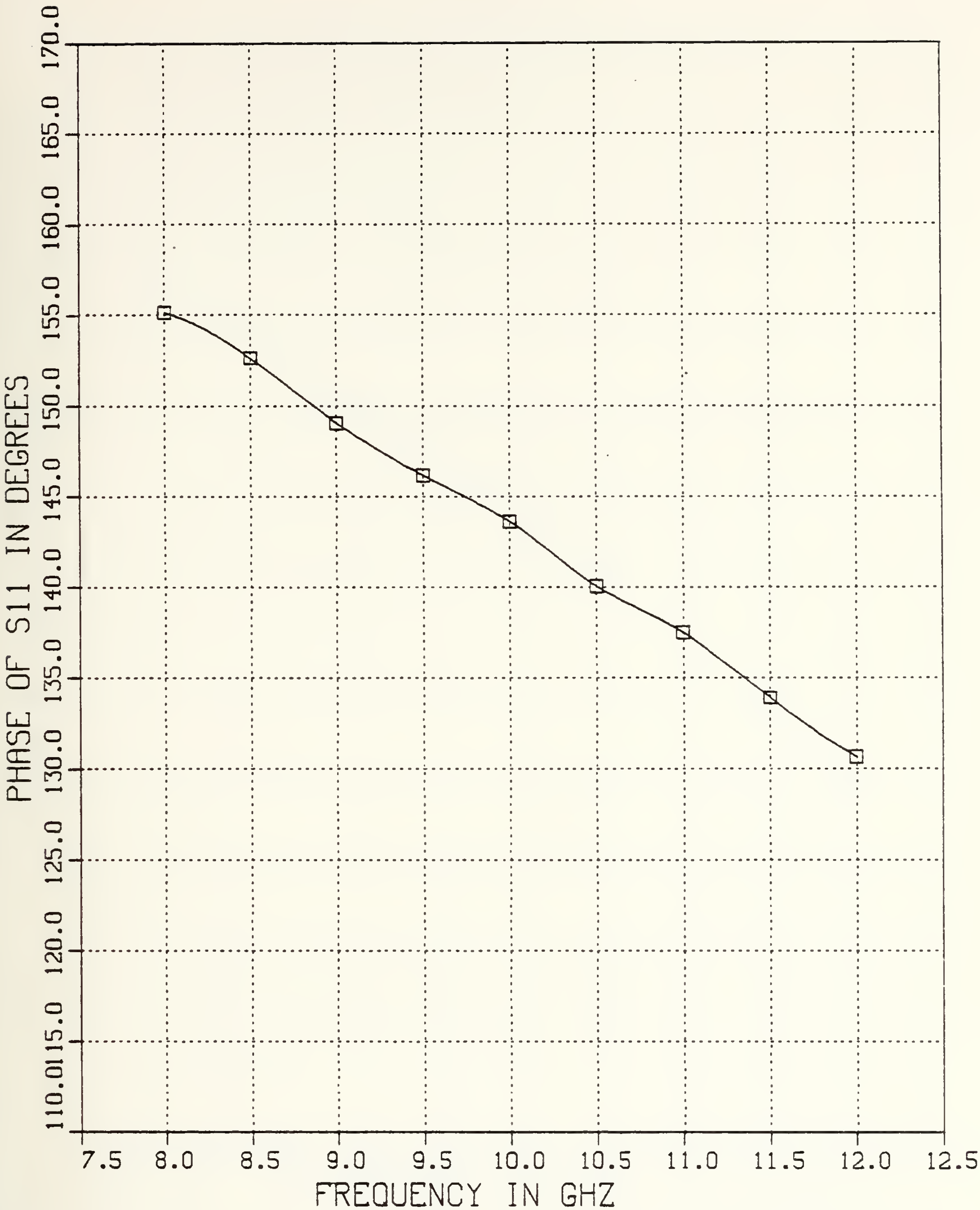


Figure F-49 θ_{11} v.s. Frequency for T=.05 inch Inductive Strip, w/b=.2, and ε_{r_2}=1.0

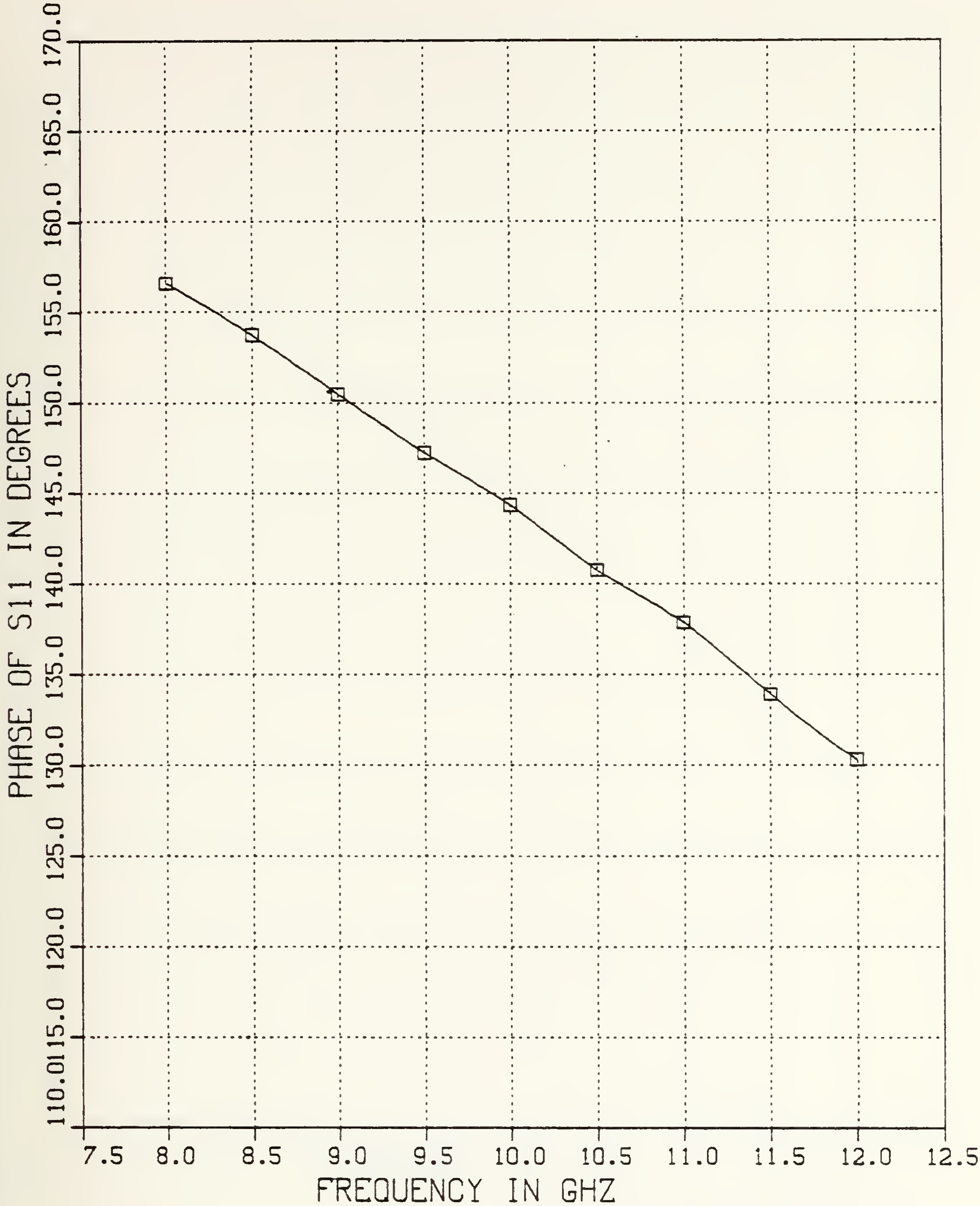


Figure F-50 θ_{11} v.s. Frequency for T=0.1 inch Inductive Strip, w/b=.2, and ε_{r_2}=1.0

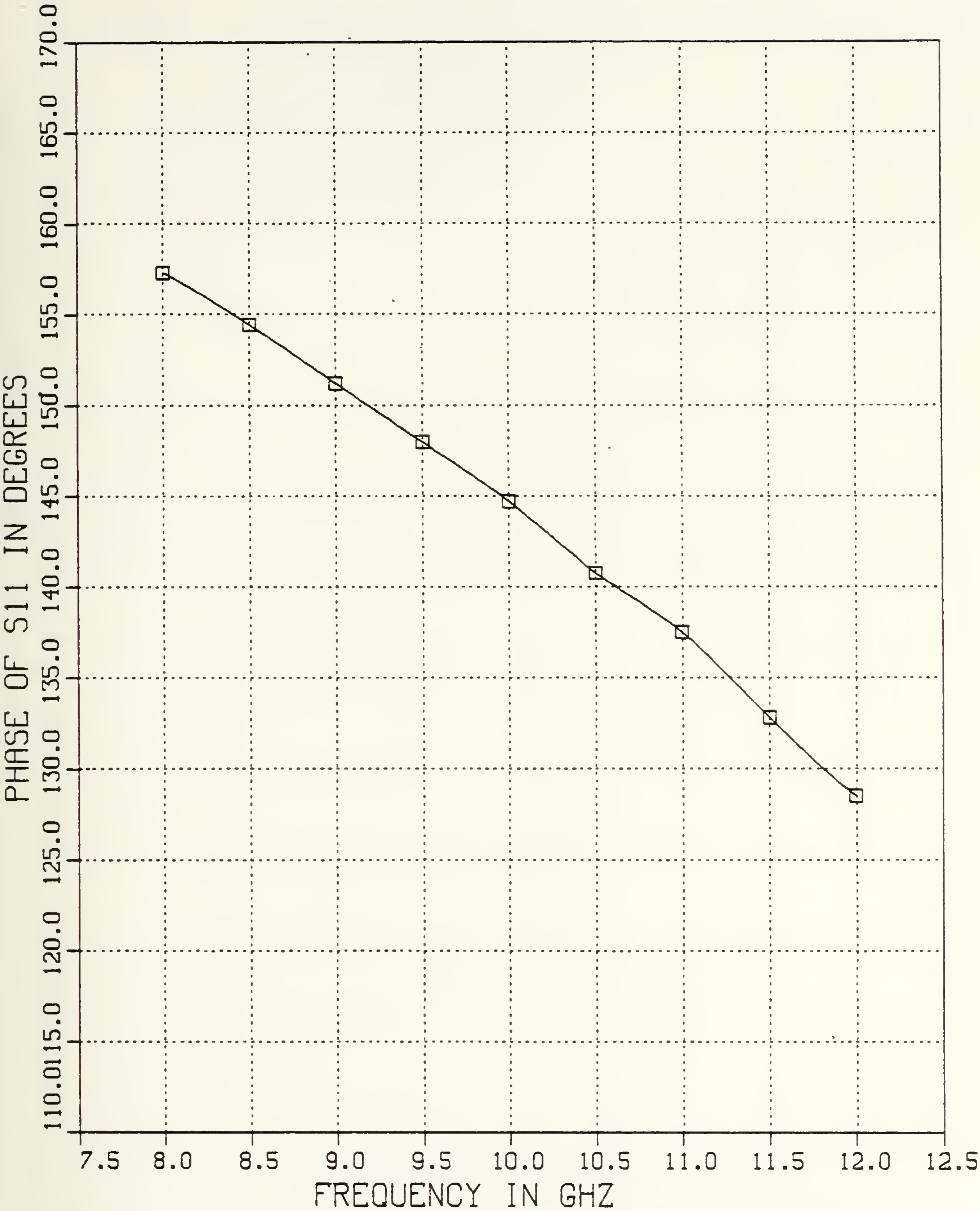


Figure F-51 θ_{11} v.s. Frequency for T=0.2 inch Inductive Strip, w/b=.2, and $\varepsilon_{r_2}=1.0$

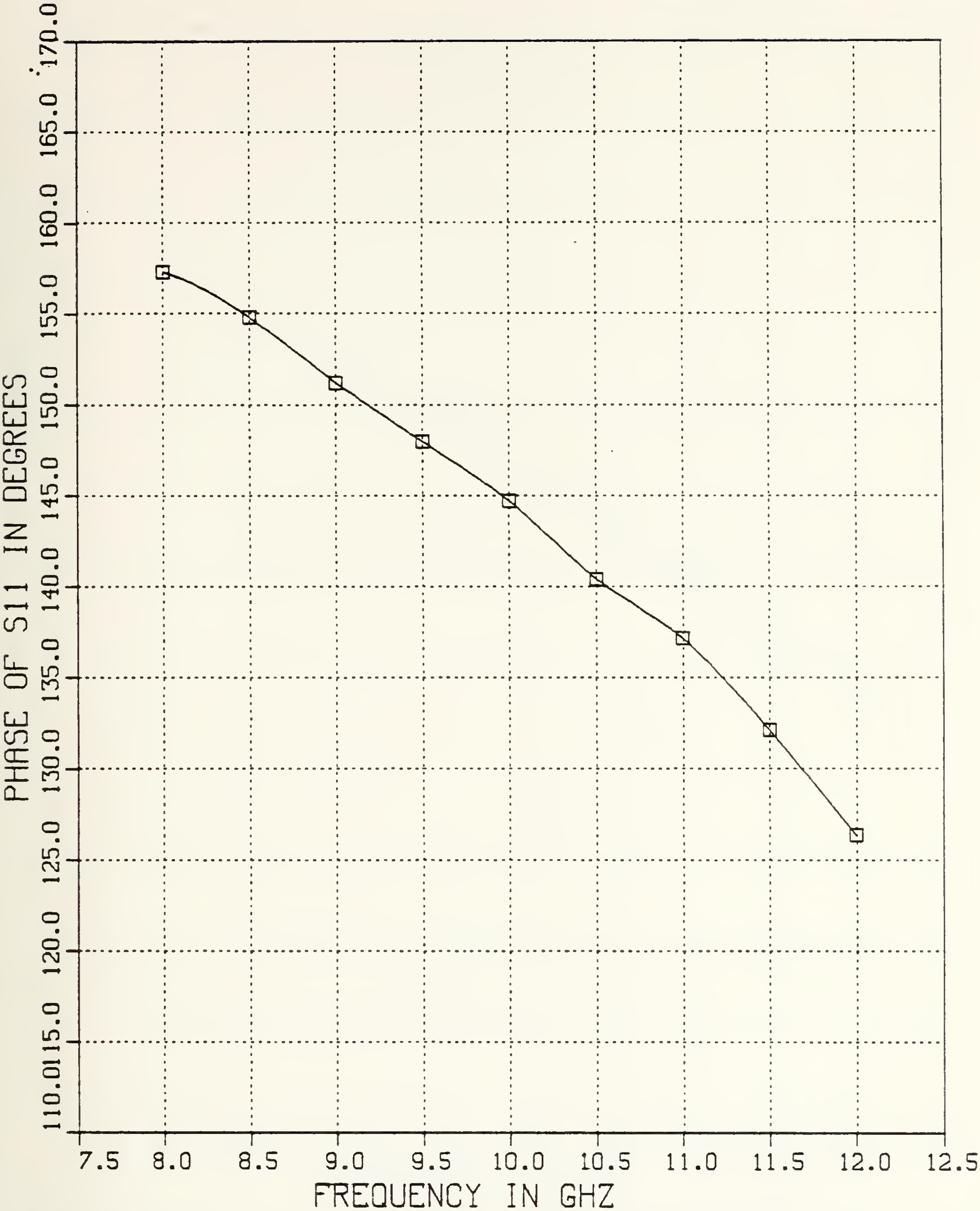


Figure F-52 θ_{11} v.s. Frequency for T=0.5 inch Inductive Strip, w/b=.2, and ε_{r_2}=1.0

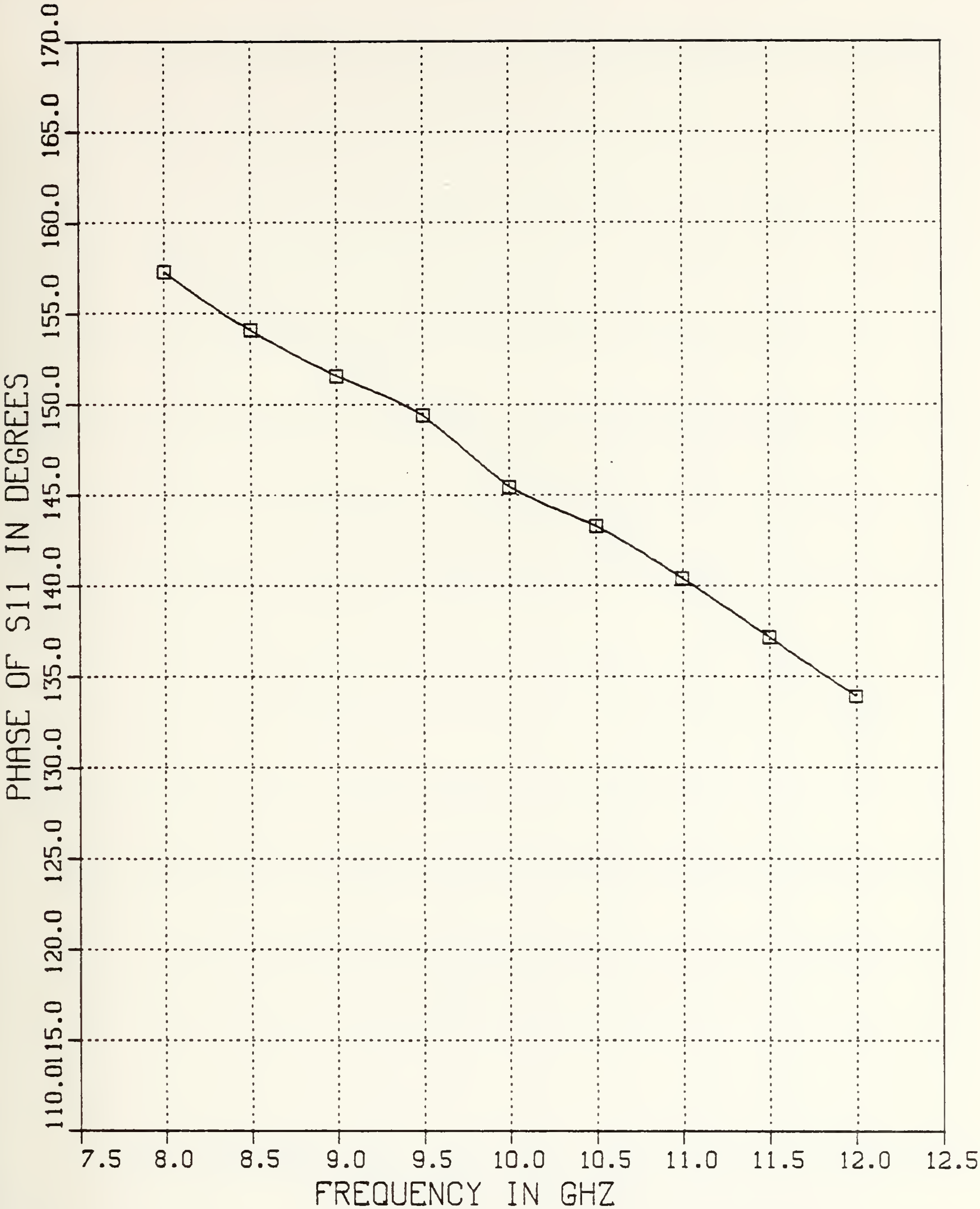


Figure F-53 θ_{11} v.s. Frequency for T=.05 inch Inductive Strip, w/b=.1, and ε_{r_b}=1.0

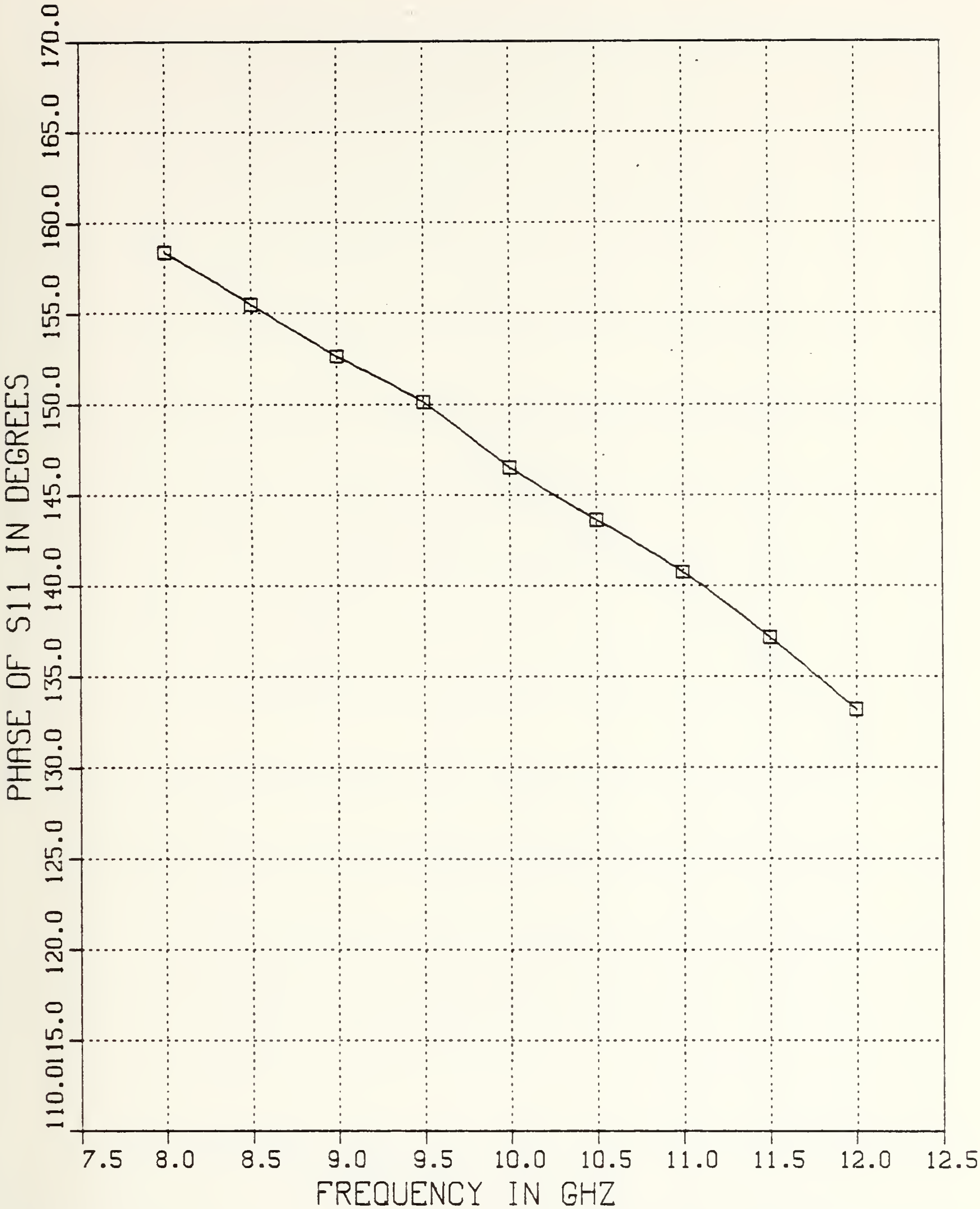


Figure F-54 θ_{11} v.s. Frequency for T=0.1 inch Inductive Strip, w/b=.1, and ε_{r_2}=1.0

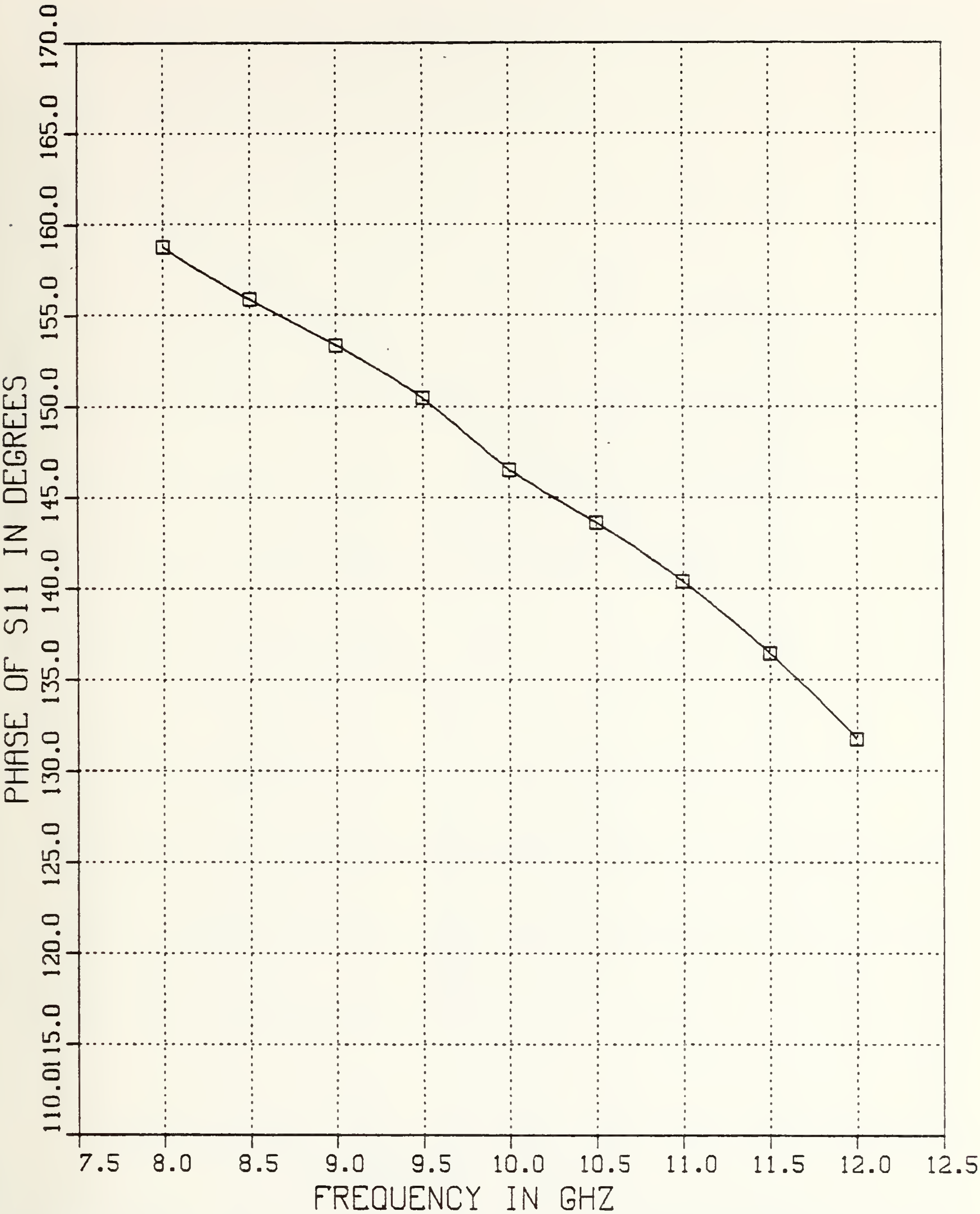


Figure F-55 θ_{11} v.s. Frequency for T=0.2 inch Inductive Strip, w/b=.1, and ε_{r_2}=1.0

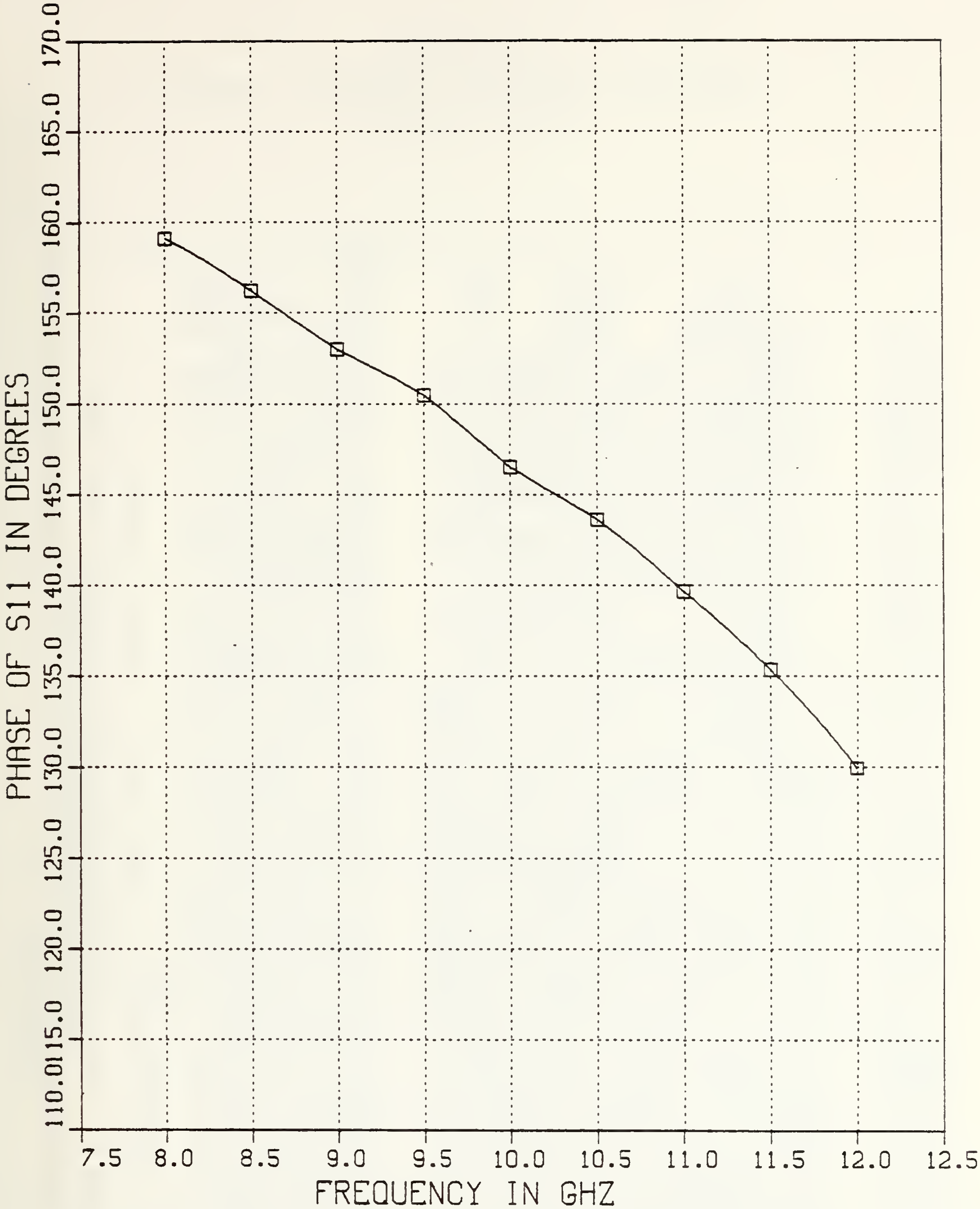


Figure F-56 θ_{11} v.s. Frequency for T=.5 inch Inductive Strip, w/b=.1, and ε_{r_2}=1.0

APPENDIX G

FINCAV: PROGRAM LISTING FOR THE SINGLE RESONANT CAVITY

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$JOB            ID,XREF,EXT
C*****************************************************************************
C               FIN-LINE TRANSMISSION STRUCTURE                              *
C       SINGLE CAVITY RESONANCE LENGTH AND EQUIVALENT REACTANCE              *
C                                                                            *
CAUTHORS: PROF. JEFFERY B. KNORR AND CPT. JOHN C. DEAL                       *
C   DATE: 30 MARCH 1984                                                      *
C                                                                            *
C      THIS ROUTINE IS DESIGNED TO CALCULATE THE FIN-LINE GUIDE WAVE-        *
C  LENGTH, RESCNANT LENGTH OF A SINGLE RESCNANT CAVITY, AND THE CORR-        *
C  ESPONDING EQUIVALENT REACTANCE OF THE SHORTING SEPTUM (INDUCTIVE          *
C  DISCONTINUITY). TJ USE THIS ROUTINE, SELECT THE FIN-LINE GEOMETRIC        *
C  PARAMETERS AND FREQUENCIES OF INTEREST AND LOAD THE DATA FILE AS          *
C  SHOWN BELOW. SELECT THE NUMBER OF BASIS FUNCTIONS DESIRED BY SET-         *
C  TING THE MATRIX ORDER EQUAL TO THE NUMBER OF BASIS FUNCTIJNS.FOR          *
C  MATICES LARGER THAN 10X10 (ORDER = 10), SET XCONST = 40.0 AND             *
C  ZCONST = 1000.0, AT LEAST. CHANGE THE ARRAY DIMENSIONS OF LL AND LT*
C  AND THE ROWDIM VARIABLE IF THE ORDER IS SELECTED TO BE GREATER THAN*
C  20.                                                                       *
C                                                                            *
C                     ALGORITHMIC STRUCTURE                                  *
C      THE PROGRAM CONSISTS OF THREE NESTED 'DO' LOOPS WITH TWO              *
C  EMBEDDED SEARCH ROUTINES AND SEVEN SUBROUTINES. THE FIRST (OUTER)         *
C  LOOP IS AN ITERATION OVER THE FIN GAP WIDTHS OF INTEREST,W/B. THE         *
C  SECOND (MIDDLE) LOOP IS AN ITERATION OVER THE FREQUENCIES OF              *
C  INTEREST,D/LAMBDA. WITHIN THIS LOOP IS A MODIFIED BISECTIONAL             *
C  SEARCH FOR THE NORMALIZED GUIDE WAVELENGTH,LAMBDA PRIME/LAMBDA,           *
C  WHICH CAUSES THE INNER PRODUCT OF THE G11 DYADIC GREENS FUNCTION          *
C  AND THE SQUARE OF THE X-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD        *
C  TO GO TO ZERO. AFTER THIS SEARCH IS THE THIRD (INNER) LOOP WHICH          *
C  IS AN ITERATION OVER THE NORMALIZED SEPTUM WIDTHS OF INTEREST,T/D.        *
C  A SIMPLE BISECTIONAL SEARCH IS USED TO FIND THE VALUE OF THE              *
C  NORMALIZED RESONANT LENGTH, L/LAMBDA PRIME, WHICH CAUSES THE              *
C  DETERMINANT OF A MATRIX OF THE INNER PRODUCT TERMS TO GO TO ZERO.         *
C  THE ELEMENTS OF THIS MATRIX CONSIST OF THE INNER PRODUCT OF THE           *
C  G11 DYADIC GREEN'S FUNCTION, THE SQUARE OF THE X-COMPONENT OF THE         *
C  X-DIRECTED ELECTRIC FIELD, THE MTH Z-COMPONENT OF THE X-DIRECTED          *
C  ELECTRIC FIELD, AND THE NTH Z-COMPONENT OF THE X-DIRECTED ELECTRIC        *
C  FIELD. THEREFORE, WITHIN THIS SEARCH THE MATRIX OF <M,N> INNER            *
C  PRODUCT TERMS IS GENERATED FOR EACH ESTIMATE OF THE SOLUTION AND          *
C  CORRESPONDINGLY EACH SEARCH ITERATION.                                    *
C                                                                            *
C                   INPUT PARAMETER DATA FILE                                *
C      THE INPUT DATA CONSISTS OF THE FIN-LINE GEOMETRIES, RELATIVE          *
C  DIELECTRIC CONSTANTS, AND FEQUENCIES OF INTEREST IN NORMALIZED            *
C  FORM AS DESCRIBED IN THE VARIABLE DEFINITIONS. THE INPUT DATA FILE        *
C  (FIELDS) IS FOUND AT THE END OF THE PROGRAM AND IS ORGANIZED AS           *
C  FOLLOWS:                                                                  *
```

```
C                                                                      *
C                  INPUT DATA FILE                                     *
C                                                                      *
C D/LAMBDA           NORMALIZED FREQUENCIES                            *
C D/LAMBDA           NORMALIZED FREQUENCIES                            *
C EPSR1,2,3          RELATIVE DIELECTRIC CONSTANTS FOR REGIONS 1,2,3   *
C H1/D, H2/D, B/D    NORMALIZED FIN POSITION COORDS AND WG HEIGTH      *
C W/B                NORMALIZED FIN GAP WIDTHS                         *
C T/D                NORMALIZED SEPTUM WIDTHS                          *
C                                                                      *
C    HERE W = FIN GAP WIDTH                                            *
C         B = HEIGTH OF RECTANGULAR WAVEGUIDE                          *
C         D = DIELECTRIC THICKNESS                                     *
C         LAMBDA = FREE SPACE WAVELENGTH                               *
C         T = WIDTH OF THE INDUCTIVE STRIP (SEPTUM)                    *
C                                                                      *
C*****VARIABLE DEFINITIONS**********************************************
C   ACCUR = ACCURACY OF THE RESULTING NUMERICAL DETERMINATION OF THE   *
C           RESONANT LENGTH.                                           *
C   ALFD = NORMALIZED 'X' TRANSFORM VARIABLE                           *
C   BETAD = NORMALIZED 'Z' TRANSFORM VARIABLE                          *
C   BOVD = WAVEGUIDE HEIGTH/D                                          *
C   BOVW = WAVEGUIDE HEIGTH/FIN GAP WIDTH                              *
C   DOVL = D/FREE SPACE WAVELENGTH                                     *
C   EPSR1,2,3 = RELATIVE DIELECTRIC CONSTANTS FO REGIONS 1,2,3 RESP.   *
C   H1OVD = FIN POSITION COORDINATE/D                                  *
C   H2OVD = FIN POSITION COORDINATE/D                                  *
C   LL = MATRIX OF INNER PRODUCT TERMS FOR THE LEFT ESTIMATE OF LRES   *
C   LOVD = RESONANT LENGTH/D                                           *
C   LPOVL = GUIDE WAVELENGTH/FREE SPACE WAVELENGTH                     *
C   LRES = ESTIMATE OF THE RESONANCE LENGTH/GUIDE WAVELENGTH           *
C   LT = MATRIX OF INNER PRODUCT TERMS FOR THE TEST ESTIMATE OF LRES   *
C   ORDER = ORDER OF THE MATRICES LL AND LT                            *
C   ROWDIM = ROW DIMENSION OF THE MATRICES LL AND LT                   *
C   TOVD = SEPTUM THICKNESS/D                                          *
C   TOVL = SEPTUM THICKNESS/RESONANT LENGTH                            *
C   WOVB = INVERSE OF BOVW                                             *
C   XCONST = CONSTANT ADJUSTING THE LIMIT OF SUMMATION OVER ALFA(N)    *
C   XLMIN = MINIMUM LIMIT OF THE SEARCH INTERVAL                       *
C   XRMAX = MAXIMUM LIMIT OF THE SEARCH INTERVAL                       *
C   XSC = SHORT CIRCUIT REACTANCE                                      *
C   ZCONST = CONSTANT ADJUSTING THE LIMIT OF SUMMATION OVER ZETA(K)    *
C   ZETAD = NORMALIZED 'Z' TRANSFORM VARIABLE                          *
C*****VARIABLE DECLARATION**********************************************
      REAL LPROVL,LERES,LORES,LRES,LR,LL
      INTEGER ORDER,ROWDIM
      DIMENSICN LL(20,20)
      DIMENSICN LR(20,20)
```

```
      DIMENSICN DOVLI(9)
      DIMENSICN WOVBI(8)
      DIMENSICN TOVDI(8)
      COMMON/C1/EPSR1,EPSR2,EPSR3,H1OVD,H2OVD,BOVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      COMMON/C4/TOVD
      COMMON/C5/XCONST,ZCONST
C     SPECIFY L(M,N) ORDER AND ROW DIMENSION
      ORDER = 3
      ROWDIM = 20
C     SPECIFY LIMIT OF SUMMATION CONSTANTS FOR THE X AND Z FIELDS
      XCONST = 22.0
      ZCONST = 100.0
C     SPECIFY MIN AND MAX SEARCH INTERVAL LIMITS,(LEFT AND RIGHT)
      XLMIN = 0.1
      XRMAX = 0.5
C     SPECIFY SEARCH ACCURACY RESIDUE
      ACCUR = 0.0001
C     READ INPUT DATA
      READ(5,101)(DOVLI(I),I=1,7)
      READ(5,102)(DOVLI(I),I=8,9)
      READ(5,100) EPSR1,EPSR2,EPSR3
      READ(5,100) H1OVD,H2OVD,BOVD
      READ(5,100) WOVBI
      READ(5,100) TOVDI
C     WRITE STRUCTURE PARAMETERS
      WRITE(6,110)
      WRITE(6,120) ORDER
      WRITE(6,200)
      WRITE(6,300) EPSR1,EPSR2,EPSR3
      WRITE(6,400) H1OVD,H2OVD,BOVD
      WRITE(6,100)
      WRITE(6,100)
      WRITE(6,100)
      WRITE(6,499)
      WRITE(6,100)
C     DEFINE CONSTANTS
      PI = ARCCS(-1.0)
      C2PI=6.283185307
      C2PISQ=C2PI**2
C     SPECIFY FIN GEOMETRY
      DO 12 IW=1,8
         WOVB=WOVBI(IW)
         WOVD=WOVB*BOVD
         IF(WOVB.EQ.0.) GO TO 13
C     SPECIFY FREQUENCY (DOVL)
         DO 11 ID=1,9
```

```
                DCVL=DOVLI(ID)
                IF(DOVL.EQ.0.) GO TO 12
                WRITE(6,500) WOVB,WOVD,DOVL
C SEARCH FOR ZERO OF IPROD1
C REF: GOTTFREID,PROGRAMMING WITH FORTRAN IV, PG. 157
C SEARCH INTERVAL IS XL=.1 TO XR=3
                XL=.1
                XR =3.
                EPSLN=.0013579
                ITER=1
C    CALCULATE INTERIOR POINTS
     1          XL1=XL+.5*(XR-XL-EPSLN)
                XR1=XL1+EPSLN
C    SUBROUTINE IPROD USES COMMON BLOCKS /C1/C2/C3/
                CALL IPROD1(XL1,YL1)
                CALL IPROD1(XR1,YR1)
                IF(YL1**2-YR1**2) 2,5,3
C    YR1**2 GREATER THAN YL1**2
     2          XR=XR1
                GC TO 4
C    YL1**2 GREATER THAN YR1**2
     3          XL=XL1
C    TEST FOR END OF SEARCH
     4          IF(ITER.GE.100) GO TO 6
                ITER=ITER+1
                IF(XR-XL.GT.3.*EPSLN) GO TO 1
     5          LPROVL=.5*(XL1+XR1)
C    WRITE SOLUTION
                WRITE(6,600) LPROVL
                GO TO 8
C    WRITE OUTPUT - SEARCH FAILED TO CONVERGE
     6          WRITE(6,610)
                GO TO 11
     8     CONTINUE
C    SEARCH FOR RESONANT LENGTH OF FINLINE CAVITY
                CC 28 IT=1,8
                   TOVD=TOVDI(IT)
                   IF(TOVD.EQ.0.) GO TO 11
C    SEARCH INTERVAL IS XLMIN TO XRMAX
                   XL = XLMIN
                   XR = XRMAX
                   EPSLN = .5*ABS(XL-XR)
                   DUM = 0.0
                   ITER=1
C   CALCULATE INTERIOR POINTS
    21             IF(.NOT.((EPSLN.GE.ACCUR).AND.(ITER.LT.100))) GO TO 22
C    TEST FOR END OF SEARCH LINE 21 ABOVE
                   XEST = .5*(XL+XR)
```

```
C   CALCULATE L(M,N) MATRIX ELEMENTS;   L(M,N) * ALFA(N) =G(M)
                 DO 31 MTH = 1,ORDER
                    DO 32 NTH = MTH,ORDER
                       CALL IPROD2(MTH,NTH,LPROVL,XL,YLEL)
                       LL(MTH,NTH) = YLEL
                       IF(.NOT.(MTH.NE.NTH)) GO TO 33
                          LL(NTH,MTH) = LL(MTH,NTH)
 33                    CONTINUE
                       CALL IPROD2(MTH,NTH,LPROVL,XEST,YREL)
                       LR(MTH,NTH) = YREL
                       IF(.NOT.(MTH.NE.NTH)) GO TO 34
                          LR(NTH,MTH) = LR(MTH,NTH)
 34                    CONTINUE
 32                 CONTINUE
 31              CONTINUE
C                DO 35 MTH = 1,ORDER
C                   WRITE(6,130) (LL(MTH,NTH),NTH=1,ORDER)
C   35           CONTINUE
C                DO 36 MTH = 1,ORDER
C                   WRITE(6,130) (LR(MTH,NTH),NTH=1,ORDER)
C   36           CONTINUE
C    CALCULATE THE DETERMINANT OF THE L(M,N) MATRIX:
                 CALL DTERM(ORDER,LL,DETL,RCWDIM)
                 YL = DETL
                 CALL DTERM(ORDER,LR,DETR,ROWDIM)
                 YR = DETR
                 DUM = YL*YR
                 IF(.NOT.(DUM.LT.0.0)) GO TO 23
                    XR = XEST
                 GO TO 24
 23              CONTINUE
                    XL = XEST
 24              CONTINUE
                 EPSLN = .5*ABS(XL-XR)
                 ITER = ITER+1
                 GO TO 21
 22              CONTINUE
                 IF(.NOT.(ITER.LT.100)) GO TO 25
                    LRES=.5*(XL+XR)
                    X=TAN(C2PI*.5*(.5-LRES))
C    WRITE SOLUTION FOR SHORT CIRCUIT REACTANCE
                    WRITE(6,800) TOVD,LRES,X
                 GO TO 26
 25              CONTINUE
C    WRITE OUTPUT - SEARCH FAILED TO CONVERGE
                    WRITE(6,805) TOVD
 26              CONTINUE
 28           CONTINUE
```

```
   11      CONTINUE
   12  CONTINUE
   13  STOP
  100  FORMAT(8F10.8)
  101  FORMAT(7F10.8)
  102  FORMAT(2F10.8)
  110  FORMAT(' THIS IS THE FINCAV PROGRAM TO DETERMINE THE RESONANT'/
     *1X,' LENGTH OF A SINGLE FIN-LINE RESONANT CAVITY, THE GUIDE'/
     *1X,' WAVELENGTH, AND THE EQUIVALENT REACTANCE OF THE SHORTING'/
     *1X,' SEPTUM (INDUCTIVE DISCONTINUITY)'/)
  120  FORMAT('THE DIMENSION OF THE MATRIX IS = ',I4,4X,' EPSLN=.0001'/)
C 130  FORMAT(1X,2F10.7/)
  200  FORMAT(' FINLINE PARAMETERS ARE'/)
  300  FORMAT(' EPSR1 = ',F4.1,5X,'EPSR2 = ',F4.1,5X,'EPSR3 = ',
     1F4.1)
  400  FORMAT(' H1/D = ',F4.1,6X,'H2/D = ',F4.1,6X,'B/D = ',F4.1)
  499  FORMAT('    W/B      ','    W/D     ','    D/L     ','  LPR/L   '
     1,'  TOVD    ',' LRES/LPR ','    XSC     ')
C 500  FORMAT FOR W/B, W/D AND D/L
  500  FORMAT(F7.3,3X,F7.3,3X,F7.4)
C 600  FORMAT FOR LPR/L
  600  FORMAT(29X,F7.3)
  610  FORMAT('+',29X,'    MSG2    '/)
C 800  FORMAT FOR T/D, LRES/LPR AND XSC
  800  FORMAT(39X,F7.3,3X,F7.3,3X,F7.3)
  805  FORMAT('+',39X,F7.3,3X,'    MSG3    '/)
C 810  FORMAT('+',39X,F7.3,3X,'    MSG4    ','    MSG4    '/)
C 820  FORMAT('+',59X,'    MSG5    '/)
C ERROR MESSAGES
C MSG2: SEARCH FOR ZERO OF IPROD1 FAILED TO CONVERGE
C MSG3: SEARCH FOR ZERO OF IPROD2 FAILED TO CONVERGE
C MSG4: SEARCH FOR ZERO OF IPROD3 FAILED TO CONVERGE
C MSG5: SEARCH FOR ZERO OF IPROD4 FAILED TO CONVERGE
       END
C SUBR***********************************************************************
$EJECT
       SUBROUTINE IPROD1(LPROVL,SUM1)
C****************************************************************************
C      THIS SUBROUTINE IS DESIGNED TO CALCULATE THE INNER PRODUCT OF   *
C THE G11 DYADIC GREEN'S FUNCTION AND THE SQUARE OF THE FOURIER TRANS*
C -FORM OF THE X-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD           *
C*****VARIABLE DEFINITIONS***************************************************
C EXXSQ = SQUARE OF THE FOURIER TRANSFORM OF THE X-COMPONENT OF THE   *
C          X-DIRECTED ELECTRIC FIELD                                   *
C G11 = THE G11 DYADIC GREEN'S FUNCTION                                *
C MMAX = THE LIMIT OF SUMMATION OVER ALFD                              *
C SUM1 = THE RETURNED VALUE OF THE INNER PRODUCT                       *
C*****VARIABLE DECLARATION***************************************************
```

```
      REAL LPROVL
      COMMON/C1/EPSR1,EPSR2,EPSR3,H1OVD,H2OVD,BOVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      COMMON/C5/XCONST,ZCONST
      BETAD = C2PI*DOVL/LPROVL
      BOVW = 1./WOVB
C    SPECIFY LIMIT FOR SUMMATION ON N
      IF(WOVB.EQ.1.) MMAX=1
      IF(WOVB.LT.1.) MMAX=INT((XCONST*BOVW)/C2PI)
      IF(MMAX.GT.40) MMAX=40
      SUM1  = 0.0
      DO 11 M=1,MMAX
         N=M-1
         ALFD=N*C2PI/BOVD
         CALL GRN11(ALFD,BETAD,DOVL,G11)
         CALL EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
         SUM1=SUM1+2.*G11*EXXSQ
         IF(N.EQ.0) SUM1=G11
   11 CONTINUE
      RETURN
      END
C SUBR******************************************************************
$EJECT
      SUBROUTINE IPROD2(MTH,NTH,LPROVL,LRES,SUM2)
C***********************************************************************
C      THIS SUBROUTINE IS DESIGNED TO CALCULATE THE INNER PRODUCT OF   *
C THE G11 DYADIC GREEN'S FUNCTION, THE FOURIER TRANSFORM OF THE MTH    *
C Z-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD, AND THE FOURIER TRANS*
C -FORM OF THE NTH Z-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD.     *
C*****VARIABLE DEFINITIONS**********************************************
C EXXSQ = SQUARE OF THE FOURIER TRANSFORM OF THE X-COMPONENT OF THE   *
C         X-DIRECTED ELECTRIC FIELD                                    *
C G11 = THE G11 DYADIC GREEN'S FUNCTION                                *
C EXZM = THE FOURIER TRANSFORM OF THE MTH Z-COMPONENT OF THE          *
C         X-DIRECTED ELECTRIC FIELD                                    *
C EXZN = THE FOURIER TRANSFORM OF THE NTH Z-COMPONENT OF THE          *
C         X-DIRECTED ELECTRIC FIELD                                    *
C MTH = ROW INDEX                                                      *
C NTH = COLUMN INDEX                                                   *
C NMAX = THE LIMIT OF SUMMATION OVER ALFD                              *
C KMAX = THE LIMIT OF SUMMATION OVER ZETAD                             *
C SUM2 = THE RETURNED VALUE OF THE INNRE                               *
C*****VARIABLE DECLARATION**********************************************
      REAL LPROVL,LRES,LOVD
      COMMON/C1/EPSR1,EPSR2,EPSR3,H1OVD,H2OVD,BOVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
```

```
      COMMON/C4/TOVD
      COMMON/C5/XCONST,ZCONST
      LOVD=LRES*LPROVL/DOVL
      TOVL=TCVD/LOVD
      BOVW = 1./WOVB
C     SPECIFY LIMIT FOR SUMMATION ON N
      IF(WOVB.EQ.1.) NMAX=0
      IF(WOVB.LT.1.) NMAX=INT((XCONST*BOVW)/C2PI)
      IF(NMAX.GT.40) NMAX=40
C     SPECIFY LIMIT FOR SUMMATION ON K
      CONST = ZCONST
      RMN = AMAXO(MTH,NTH)
      KMAX=INT(1.5 + (1.+TOVL)*SQRT(CONST+((2.*RMN-1.)/2.)**2))
      SUM2=0.0
      N=0
      DO 10 K=1,KMAX
         ALFD=N*C2PI/BOVD
         ZETAD=((K-.5)*C2PI)/((1.+TOVL)*LOVD)
         BETAD=ZETAD
         CALL GRN11(ALFD,BETAD,DOVL,G11)
         CALL EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
         CALL EXZ(MTH,ZETAD,LRES,LPROVL,EXZFM)
         EXZM = EXZFM
         CALL EXZ(NTH,ZETAD,LRES,LPROVL,EXZFN)
         EXZN = EXZFN
         SUM2=SUM2+2.*G11*EXXSQ*EXZM*EXZN
   10 CONTINUE
      IF(NMAX.EQ.0) GO TO 14
      DO 13 N=1,NMAX
         DO 12 K=1,KMAX
            ALFD=N*C2PI/BOVD
            ZETAD=((K-.5)*C2PI)/((1.+TOVL)*LCVD)
            BETAD=ZETAD
            CALL GRN11(ALFD,BETAD,DOVL,G11)
            CALL EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
            CALL EXZ(MTH,ZETAD,LRES,LPROVL,EXZFM)
            EXZM = EXZFM
            CALL EXZ(NTH,ZETAD,LRES,LPROVL,EXZFN)
            EXZN = EXZFN
            SUM2=SUM2+4.*G11*EXXSQ*EXZM*EXZN
   12    CONTINUE
   13 CONTINUE
   14 CONTINUE
      RETURN
      END
C SUBR*****************************************************************
$EJECT
      SUBROUTINE GRN11(ALFD,BETAD,DOVL,G11)
```

```
C*******************************************************************
C      THIS SUBROUTINE IS DESIGNED TO CALCULATE THE G11 DYADIC GREEN'S*
C FUNCTION                                                          *
C*****VARIABLE DEFINITIONS******************************************
C G11 = THE G11 DYADIC GREEN'S FUNCTION, RETURNED VALUE             *
C G1DSQ                                                             *
C G2DSQ                                                             *
C G3DSQ                                                             *
C KC1DSQ                                                            *
C KC2DSQ                                                            *
C KC3DSQ                                                            *
C WEPS1D                                                            *
C WEPS2D                                                            *
C WEPS3D                                                            *
C WMUD                                                              *
C*****VARIABLE DECLARATION******************************************
      COMMON/C1/EPSR1,EPSR2,EPSR3,H1OVD,H2OVD,BOVD
      COMMON/C2/C2PI,C2PISQ,PI
      REAL KC1DSQ,KC2DSQ,KC3DSQ,LPROVL,LOVLPR
C    CALCULATE VARIABLES DEPENDENT ON FREQUENCY
      WMUD=60.*C2PISQ*DOVL
      WEPS1D=EPSR1*DOVL/60.
      WEPS2D=EPSR2*DOVL/60.
      WEPS3D=EPSR3*DOVL/60.
C    CALCULATE VARIABLES DEPENDENT ON FRRQUENCY AND BETAD
      BETDSQ=BETAD**2
      KC1DSQ=C2PISQ*EPSR1*DOVL**2-BETDSQ
      KC2DSQ=C2PISQ*EPSR2*DOVL**2-BETDSQ
      KC3DSQ=C2PISQ*EPSR3*DOVL**2-BETDSQ
C    CALCULATE VARIABLES DEPENDENT ON FREQUENCY, BETAD AND ALFD
      ALFDSQ=ALFD**2
      G1DSQ=ALFDSQ-KC1DSQ
      G2DSQ=ALFDSQ-KC2DSQ
      G3DSQ=ALFDSQ-KC3DSQ
      CALL TFN(G1DSQ,H1OVD,TFN1)
      CALL TFN(G2DSQ,1.,TFN2)
      CALL TFN(G3DSQ,H2OVD,TFN3)
      D11=-KC2DSQ*(1.+(WEPS3D*G3DSQ*KC2DSQ*TFN2)/(WEPS2D*G2DSQ*KC3DSQ*
     1TFN3))
      D12=((ALFD*BETAD)/(WEPS2D*G2DSQ))*((KC2DSQ/KC1DSQ)-1.)*KC2DSQ
      D21=-ALFD*BETAD*((KC2DSQ/KC1DSQ)+((WEPS3D*KC2DSQ*G3DSQ*TFN2)/
     1(WEPS2D*KC3DSQ*G2DSQ*TFN3)))
      D22=WMUD*(1.+((KC2DSQ*TFN3)/(KC3DSQ*TFN2))+((ALFDSQ*BETDSQ)/(G2DSQ
     1*WMUD*WEPS2D))*((KC2DSQ/KC3DSQ)-1.))
      DET=D11*D22-D21*D12
C    CALCULATE G11
      G11A=-KC1DSQ/(WMUD*TFN1)
      G11B=-(KC2DSQ*D12*ALFD*BETAD*KC2DSQ*TFN2)/(DET*WMUD*G2DSQ*KC3DSQ)
```

```
      G11C=(KC2DSQ*D12*ALFD*BETAD*TFN2)/(DET*WMUD*G2DSQ)
      G11D=-(KC2DSQ*D11*KC2DSQ*TFN3)/(DET*KC3DSQ*G2DSQ)
      G11E=-(KC2DSQ*D11)/(DET*TFN2)
      G11=G11A+G11B+G11C+G11D+G11E
      RETURN
      END
C SUBR*************************************************************
$EJECT
      SUBROUTINE TFN(GIDSQ,HIOVD,TFNI)
C******************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE TANGENT AND THE    *
C HYPERBOLIC TANGENT FUNCTIONS FOR THE GRN11 SUBROUTINE.              *
C*****VARIABLE DEFINITIONS*****************************************
C GID                                                                 *
C GIDSQ                                                               *
C HIOVD                                                               *
C TFNI = THE RETURNED VALUE                                           *
C*****VARIABLE DECLARATION*****************************************
      GID=SQRT(ABS(GIDSQ))
      ARG=GIDSQ*HIOVD**2
      IF(ARG.LE.0.) GO TO 1
      IF((ARG.GT.0.).AND.(ARG.LT.100.)) GO TO 2
      IF(ARG.GE.100.) GO TO 3
    1 TFNI=-GID*TAN(GID*HIOVD)
      GO TO 4
    2 TFNI=GID*TANH(GID*HIOVD)
      GO TO 4
    3 TFNI=GID
    4 RETURN
      END
C SUBR*************************************************************
$EJECT
      SUBROUTINE EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
C******************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE FOURIER TRANSFORM *
C OF THE X-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD AND SQUARE IT. *
C*****VARIABLE DEFINITIONS*****************************************
C EXXSQ = SQUARE OF THE FOURIER TRANSFORM OF THE X-COMPONENT OF THE   *
C         X-DIRECTED ELECTRIC FIELD, THE RETURNED VALUE               *
C*****VARIABLE DECLARATION*****************************************
      COMMON/C2/C2PI,C2PISQ,PI
      WOVD=WOVB*BOVD
C     CALCULATE EXXSQ, THE X TRANSFORM OF THE E FIELD BETWEEN THE FINS
      IF(ALFD.EQ.0.) EXXSQ=1.
      IF(ALFD.GT.0.) EXXSQ=(SIN(.5*ALFD*WOVD)/(.5*ALFD*WOVD))**2
      RETURN
      END
C SUBR*************************************************************
```

```
$EJECT
      SUBROUTINE EXZ(MNTH,ZETAD,LRES,LPROVL,EXZFMN)
C**********************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE FOURIER TRANSFORM *
C OF THE Z-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD.              *
C*****VARIABLE DEFINITIONS*********************************************
C EXZFMN = THE FOURIER TRANSFORM OF THE MNTH Z-COMPONENT OF THE      *
C          X-DIRECTED ELECTRIC FIELD, THE RETURNED VALUE.            *
C MNTH = ROW OR COLUMN INDEX                                         *
C*****VARIABLE DECLARATION*********************************************
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      REAL LRES,LPROVL,LOVD,NUMER,DENOM,DIM
      LOVD=LRES*LPROVL/DOVL
      THETA=ZETAD*.5*LOVD
      ATHETA = ABS(THETA)
      DIM = FLOAT(MNTH)
      RESID = ABS(ATHETA - (((2.*DIM-1.)*PI)/2.))
      SIGN = (-1.0)**MNTH
      NUMER = COS(ATHETA)
      DENOM = ((THETA)**2) - (((((2.*DIM)-1.)*PI)/2.)**2)
      IF(.NOT.(RESID.LT.0.0000001)) GO TO 35
         EXZFMN = 1./PI
      GO TO 36
   35 CONTINUE
         EXZFMN = (SIGN)*((2.*DIM)-1.0)*(NUMER/DENOM)
   36 CONTINUE
      RETURN
      END
C  SUBR*****************************************************************
C   A. IDENTIFICATION:                                                *
C      TITLE:        DETERMINANT OF A REAL MATRIX                     *
C      CATEGORY:     DETERMINANTS                                     *
C      PROGRAMMER: JEAN BOW                                           *
C      DATE:         FEBRUARY 1965, CONVERTED TO SYSTEM/360, JANUARY 1967 *
C                    UPDATED BY P.J. COLLINS JANUARY 1984             *
C   B. PURPOSE:                                                       *
C         THIS SUBROUTINE COMPUTES THE DETERMINANT OF A MATRIX OF REAL *
C         NUMBERS BY GAUSS' METHOD OF ELIMINATION WITH ROW PIVOTING.  *
C   C. USAGE:                                                         *
C      1. CALLING SEQUENCE:                                           *
C         CALL DTERM(N,A,DET,M)                                       *
C      2. PARAMETERS:                                                 *
C         N - ORDER OF MATRIX A. (INTEGER)                            *
C         A - MATRIX, THE DETERMINANT OF WHICH IS TO BE COMPUTED.(REAL*8)
C         DET - DETERMINANT OF MATRIX A. (REAL*8)                     *
C         M - ROW DIMENSION OF MATRIX A. (THE INTEGER APPEARING IN USER'S
C             DIMENSION STATEMENT.)                                   *
```

```
C   D. CAUTIONS TO USER:                                                    *
C      IN THE CALLING PROGRAM, "A" MUST APPEAR IN A DIMENSION STATEMENT.*
C      MATRIX A IS DESTROYED.                                              *
C   E. REFERENCES:                                                          *
C      1. KUNZ, K. S., "NUMERICAL ANALYSIS", MCGRAW-HILL, NEW YORK, 1957*
C      2. FADDEVA, V. A., "COMPUTATIONAL METHODS OF LINEAR ALGEBRA",      *
C         TRANSLATED BY CURTIS D. BENSTER, DOVER PUBLICATIONS, INC.,      *
C         NEW YORK, 1959.                                                  *
C***********************************************************************
$EJECT
      SUBROUTINE DTERM(N,A,D,M)
      IMPLICIT REAL (A-H), REAL (O-Z)
      DIMENSION A(M,M)
      DD=1.0
   10 DO 34 L=1,N
         KP=0
         Z=0.0
         DO 12 K=L,N
            IF(Z-ABS(A(K,L)))11,12,12
   11       Z=ABS(A(K,L))
            KP=K
   12    CONTINUE
         IF(L-KP)13,20,20
   13    DO 14 J=L,N
            Z=A(L,J)
            A(L,J)=A(KP,J)
   14    A(KP,J)=Z
         DD=-DD
   20    IF(L-N)31,40,40
   31    LP1=L+1
         DO 34 K=LP1,N
         IF(A(K,L))32,34,32
   32    RATIO=A(K,L)/A(L,L)
         DO 33 J=LP1,N
   33    A(K,J)=A(K,J)-RATIO*A(L,J)
   34 CONTINUE
   40 DO 41 K = 1,N
   41 DD=DD*A(K,K)
      D=DD
      RETURN
      END
$ENTRY
.0677333  .0719667  .0762     .0804333  .0846667  .0889     .0931333
.0973667  .1016
1.        1.        1.
4.5       3.5       4.0
1.0       0.5       0.1
2.0       4.0       8.0       12.0      16.0      24.0
$END
```

APPENDIX H

FINSTRP: PROGRAM LISTING FOR THE COUPLED RESONANT CAVITIES

```
$JOB              ID,XREF,EXT
C*****************************************************************************
C                  FIN-LINE TRANSMISSION STRUCTURE                           *
C       ODD AND EVEN MODE RESONANT LENGTHS OF TWO UNILATERALLY               *
C       COUPLED CAVITIES AND EQUIVALENT CIRCUIT REACTANCES AND               *
C       SCATTERING PARAMETERS FOR THE INDUCTIVE POST, (STRIP).               *
C                                                                            *
CAUTHORS: PROF. JEFFERY B. KNORR AND CPT. JOHN C. DEAL                       *
C    DATE: 30 MARCH 1984                                                     *
C                                                                            *
C       THIS ROUTINE IS DESIGNED TO CALCULATE THE FIN-LINE GUIDE WAVE-       *
C  LENGTH, THE ODD AND EVEN MODE RESONANT LENGTHS OF TWO UNILATERALLY        *
C  COUPLED FIN-LINE RESONANT CAVITIES AND THE CORRESPONDING EQUIVALENT*
C  CIRCUIT REACTANCES OF THE INDUCTIVE STRIP DISCONTINUITY AND THE           *
C  SCATTERING PARAMETERS OF THE INDUCTIVE STRIP AS FUNCTIONS OF THE          *
C  ODD AND EVEN MODE RESONANT LENGTHS BY TREATING THE STRIP AS A LOSS        *
C  -LESS TWO-PORT NETWORK. TO USE THIS ROUTINE, SELECT THE FINLINE           *
C  GEOMETRIC PARAMETERS AND THE FREQUENCIES OF INTEREST AND LOAD THE         *
C  DATA FILE AS SHOWN BELOW. SELECT THE NUMBER OF BASIS FUNCTIONS            *
C  DESIRED BY SETTING THE MATRIX ORDER EQUAL TO THE NUMBER OF BASIS          *
C  FUNCTIONS. CHANGE THE ARRAY DIMENSIONS OF LRE, LLE, LRO, AND LLO          *
C  AND THE ROWDIM VARIABLE IF THE ORDER IS SELECTED TO BE GREATER THAN*
C  20. TO OBTAIN THE ODD AND EVEN MODE RESONANT LENGTHS AS OUTPUT,           *
C  COMMENT OUT THE SCATTERING PARAMETER CALCULATIONS AND ASSOCIATED          *
C  WRITE AND FORMAT STATEMENTS. TO OBTAIN THE SCATTERING PARAMETERS AS*
C  OUTPUT, COMMENT OUT THE EQUIVALENT CIRCUIT REACTANCE CALCULATIONS         *
C  AND ASSOCIATED WRITE AND FORMAT STATEMENTS.                               *
C                                                                            *
C                       ALGORITHMIC STRUCTURE                                *
C                                                                            *
C       THE PROGRAM CONSISTS OF THREE NESTED 'DO' LOOPS WITH TWO             *
C  EMBEDDED SEARCH ROUTINES AND SEVEN SUBROUTINES. THE FIRST (OUTER)         *
C  LOOP IS AN ITERATION OVER THE FIN GAP WIDTHS OF INTEREST,W/B. THE         *
C  SECOND (MIDDLE) LOOP IS AN ITERATION OVER THE FREQUENCIES OF              *
C  INTEREST,D/LAMBDA. WITHIN THIS LOOP IS A MODIFIED BISECTIONAL             *
C  SEARCH FOR THE NORMALIZED GUIDE WAVELENGTH,LAMBDA PRIME/LAMBDA,           *
C  WHICH CAUSES THE INNER PRODUCT OF THE G11 DYADIC GREENS FUNCTION          *
C  AND THE SQUARE OF THE X-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD        *
C  TO GO TO ZERO. AFTER THIS SEARCH IS THE THIRD (INNER) LOOP WHICH          *
C  IS AN ITERATION OVER THE NORMALIZED SEPTUM WIDTHS OF INTEREST,T/D.        *
C  WITHIN THIS THIRD LOOP ARE TWO SUCCESSIVE SEARCHES FOR THE EVEN AND*
C  ODD MODE RESONANT LENGTHS, RESPECTIVELY. FOR THE EVEN AND ODD CASES*
C  A SIMPLE BISECTIONAL SEARCH IS USED TO FIND THE VALUE OF THE EVEN/        *
C  ODD NORMALIZED RESONANT LENGTH, LEO/LAMBDA PRIME, WHICH CAUSES THE        *
C  DETERMINANT OF A MATRIX OF THE INNER PRODUCT TERMS TO GO TO ZERO.         *
C  THE ELEMENTS OF THIS MATRIX CONSIST OF THE INNER PRODUCT OF THE           *
C  G11 DYADIC GREEN'S FUNCTION, THE SQUARE OF THE X-COMPONENT OF THE         *
C  X-DIRECTED ELECTRIC FIELD, THE MTH Z-COMPONENT OF THE X-DIRECTED          *
```

```
C ELECTRIC FIELD, THE NTH Z-COMPONENT OF THE X-DIRECTED ELECTRIC, AND*
C THE MTH AND NTH SPATIAL SHIFT FUNCTIONS FOR THE Z-COMPONENT FUNCT- *
C IONS. THEREFORE, WITHIN THIS SEARCH THE MATRIX CF <M,N> INNER      *
C PRODUCT TERMS IS GENERATED FOR EACH ESTIMATE OF THE SOLUTION AND   *
C CORRESPONDINGLY EACH SEARCH ITERATION. THE EQUIVALENT CIRCUIT REACT*
C -ANCES AND SCATTERING PARAMETERS ARE THEN CALCULATED FROM THE EVEN *
C ODD MODE RESONANT LENGTHS.                                         *
C                                                                    *
C                INPUT PARAMETER DATA FILE                           *
C     THE INPUT DATA CONSISTS OF THE FIN-LINE GEOMETRIES, RELATIVE   *
C DIELECTRIC CONSTANTS, AND FEQUENCIES OF INTEREST IN NORMALIZED     *
C FORM AS DESCRIBED IN THE VARIABLE DEFINITIONS. THE INPUT DATA FILE *
C (FIELDS) IS FOUND AT THE END OF THE PROGRAM AND IS ORGANIZED AS    *
C FOLLOWS:                                                           *
C                                                                    *
C                INPUT DATA FILE                                     *
C                                                                    *
C D/LAMBDA              NORMALIZED FREQUENCIES                       *
C D/LAMBDA              NORMALIZED FREQUENCIES                       *
C EPSR1,2,3             RELATIVE DIELECTRIC CONSTANTS FOR REGIONS 1,2,3  *
C H1/D, H2/D, B/D       NORMALIZED FIN POSITION CCORDS AND WG HEIGTH *
C W/B                   NORMALIZED FIN GAP WIDTHS                    *
C T/D                   NORMALIZED SEPTUM WIDTHS                     *
C                                                                    *
C     HERE W = FIN GAP WIDTH                                         *
C          B = HEIGTH OF RECTANGULAR WAVEGUIDE                       *
C          D = DIELECTRIC THICKNESS                                  *
C          LAMBDA = FREE SPACE WAVELENGTH                            *
C          T = WIDTH OF THE INDUCTIVE STRIP (SEPTUM)                 *
C                                                                    *
C*****VARIABLE DEFINITIONS********************************************
C    ACCUR = ACCURACY OF THE RESULTING NUMERICAL DETERMINATION OF THE *
C            ODC AND EVEN MODE RESONANT LENGTHS                      *
C    ALFD = NORMALIZED 'X' TRANSFORM VARIABLE                        *
C    BETAD = NORMALIZED 'Z' TRANSFORM VARIABLE                       *
C    BOVD = WAVEGUIDE HEIGTH/D                                       *
C    BOVW = WAVEGUIDE HEIGTH/FIN GAP WIDTH                           *
C    DOVL = D/FREE SPACE WAVELENGTH                                  *
C    EPSR1,2,3 = RELATIVE DIELECTRIC CONSTANTS FO REGIONS 1,2,3 RESP. *
C    H1OVD = FIN POSITION COORDINATE/D                               *
C    H2OVD = FIN POSITION COORDINATE/D                               *
C    LLE = MATRIX OF INNER PRODUCT TERMS FOR THE LEFT EST OF LERES   *
C    LLO = MATRIX OF INNER PRODUCT TERMS FOR THE LEFT EST OF LORES   *
C    LRE = MATRIX OF INNER PRODUCT TERMS FOR THE RIGHT EST OF LERES  *
C    LRO = MATRIX OF INNER PRODUCT TERMS FOR THE RIGHT EST OF LORES  *
C    LOVD = RESONANT LENGTH/D                                        *
C    LPOVL = GUIDE WAVELENGTH/FREE SPACE WAVELENGTH                  *
C    LERES = EST OF THE EVEN MODE RESONANCE LENGTH/GUIDE WAVELENGTH  *
```

```
C     LORES = EST OF THE ODD MODE RESONANCE LENGTH/GUIDE WAVELENGTH      *
C     LT = MATRIX OF INNER PRODUCT TERMS FOR THE TEST ESTIMATE OF LRES  *
C     ORDER = ORDER OF THE MATRICES LL AND LT                           *
C     ROWDIM = ROW DIMENSION OF THE MATRICES LL AND LT                  *
C     TOVD = SEPTUM THICKNESS/D                                         *
C     TOVL = SEPTUM THICKNESS/RESONANT LENGTH                           *
C     WOVB = INVERSE OF BOVW                                            *
C     XCONST = CONSTANT ADJUSTING THE LIMIT OF SUMMATION OVER ALFA(N)   *
C     XE = EVEN MODE EQUIVALENT REACTANCE                               *
C     XLEMIN = MINIMUM LIMIT OF THE SEARCH INTERVAL FOR THE EVEN MODE   *
C     XLOMIN = MINIMUM LIMIT OF THE SEARCH INTERVAL FOR THE ODD MODE    *
C     XM = MUTUAL INDUCTIVE EQUIVALENT CIRCUIT REACTANCE                *
C     XO = ODD  MODE EQUIVALENT REACTANCE                               *
C     XREMAX = MAXIMUM LIMIT OF THE SEARCH INTERVAL FOR THE EVEN MODE   *
C     XROMAX = MAXIMUM LIMIT OF THE SEARCH INTERVAL FOR THE ODD MODE    *
C     XS = EQUIVALENT CIRCUIT REACTANCE                                 *
C     ZCONST = CONSTANT ADJUSTING THE LIMIT OF SUMMATION OVER ZETA(K)   *
C     ZETAD = NORMALIZED 'Z' TRANSFORM VARIABLE                         *
C*****VARIABLE DECLARATION**************************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      REAL*8 LPROVL,LERES,LORES,LRES,LLE,LLO,LRE,LRO,MAGS11,MAGS12
      INTEGER ORDER,ROWDIM
      DIMENSION LRE(20,20)
      DIMENSION LLE(20,20)
      DIMENSION LRO(20,20)
      DIMENSION LLO(20,20)
      DIMENSION DOVLI(9)
      DIMENSION WOVBI(8)
      DIMENSION TOVDI(8)
      COMMON/C1/EPSR1,EPSR2,EPSR3,H1OVD,H2OVD,BOVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      COMMON/C4/TOVD
      COMMON/C5/XCONST,ZCONST
C     SPECIFY L(M,N) ORDER AND ROW DIMENSION
      ORDER = 4
      ROWDIM = 20
C     SPECIFY LIMIT OF SUMMATION CONSTANTS FOR THE X AND Z FIELDS
      XCONST = 40.0
      ZCONST = 250.0
C     SPECIFY MIN AND MAX SEARCH INTERVAL LIMITS,(LEFT AND RIGHT)
C       FOR THE EVEN AND ODD MODES, RESPECTIVELY
      XLEMIN = 0.1
      XREMAX = 0.5
      XLOMIN = 0.1
      XROMAX = 0.5
C     SPECIFY SEARCH ACCURACY RESIDUE
      ACCUR = 1.0D-4
```

```
C     READ INPUT DATA
         READ(5,101)(DOVLI(I),I=1,7)
         READ(5,102)(DOVLI(I),I=8,9)
         READ(5,100) EPSR1,EPSR2,EPSR3
         READ(5,100) H1OVD,H2OVD,BOVD
         READ(5,100) WOVBI
         READ(5,100) TOVDI
C     WRITE STRUCTURE PARAMETERS
         WRITE(6,110)
         WRITE(6,120) ORDER
         WRITE(6,200)
         WRITE(6,300) EPSR1,EPSR2,EPSR3
         WRITE(6,400) H1OVD,H2OVD,BOVD
         WRITE(6,100)
         WRITE(6,100)
         WRITE(6,100)
         WRITE(6,489)
         WRITE(6,100)
C     DEFINE CONSTANTS
         PI = DARCOS(-1.0D0)
         C2PI = 6.283185307
         C2PISQ = C2PI**2
C     SPECIFY FIN GEOMETRY
         DO 12  IW=1,8
              WOVB=WOVBI(IW)
              WOVC=WOVB*BOVD
              IF(WOVB.EQ.0.) GO TO 13
C     SPECIFY FREQUENCY (DOVL)
              DO  11 ID = 1,9
                   DOVL = DOVLI(ID)
                   IF(DOVL.EQ.0.) GO TO 12
                   WRITE(6,500) WOVB,WOVD,DOVL
C  SEARCH FOR ZERO OF IPROD1
C  REF: GOTTFREID,PROGRAMMING WITH FORTRAN IV, PG. 157
C  SEARCH INTERVAL IS XL=.1 TO XR=3
                   XL=.1
                   XR =3.
                   EPSLN = 1.35790D-3
                   ITER=1
C     CALCULATE INTERIOR POINTS
      1            XL1=XL+.5*(XR-XL-EPSLN)
                   XR1=XL1+EPSLN
C     SUBROUTINE IPROD USES COMMON BLOCKS /C1/C2/C3/
                   CALL IPROD1(XL1,YL1)
                   CALL IPROD1(XR1,YR1)
                   IF(YL1**2-YR1**2) 2,5,3
C     YR1**2 GREATER THAN YL1**2
      2            XR=XR1
```

```
            GC TO 4
C    YL1**2 GREATER THAN YR1**2
     3      XL=XL1
C    TEST FOR END OF SEARCH
     4      IF(ITER.GE.100) GO TO 6
            ITER=ITER+1
            IF(XR-XL.GT.3.*EPSLN) GO TO 1
     5      LPROVL=.5*(XL1+XR1)
C    WRITE SOLUTION
            WRITE(6,600) LPROVL
            GC TO 8
C    WRITE OUTPUT - SEARCH FAILED TO CONVERGE
     6      WRITE(6,610)
            GC TO 11
     8    CONTINUE
C SEARCH FOR EVEN MODE RESONANT LENGTH OF INDUCTIVELY COUPLED FINLINE
C CAVITY.
          WRITE(6,499)
          DO 28 IT = 1,8
            TOVD=TOVDI(IT)
            TOVW = TOVD/WOVD
            IF(TOVD.EQ.0.0) GO TO 11
C SEARCH INTERVAL IS XLEMIN TO XREMAX
            XLE = XLEMIN
            XRE = XREMAX
            EPSLNE = .5*DABS(XLE-XRE)
            SIGNE = 0.0
            ITERE = 1
C   CALCULATE INTERIOR POINTS
C   AND TEST FOR END OF SEARCH
    31         IF(.NOT.((EPSLNE.GE.ACCUR).AND.(ITERE.LT.100))) GO TO 32
               XRESTE = .5*(XLE+XRE)
C   CALCULATE L(M,N) MATRIX ELEMENTS;   L(M,N) * ALFA(N) =G(M)
               DO 51 MTH = 1,ORDER
                  DO 52 NTH = MTH,ORDER
                     CALL IPROD3(MTH,NTH,LPROVL,XLE,YLELE)
                     LLE(MTH,NTH) = YLELE
                     IF(.NOT.(MTH.NE.NTH)) GO TO 53
                        LLE(NTH,MTH) = LLE(MTH,NTH)
    53               CONTINUE
                     CALL IPROD3(MTH,NTH,LPROVL,XRESTE,YRELE)
                     LRE(MTH,NTH) = YRELE
                     IF(.NOT.(MTH.NE.NTH)) GO TO 54
                        LRE(NTH,MTH) = LRE(MTH,NTH)
    54               CONTINUE
    52            CONTINUE
    51         CONTINUE
C              DO 55 MTH = 1,ORDER
```

```
C                WRITE(6,130) (LLE(MTH,NTH),NTH=1,ORDER)
C   55        CONTINUE
C             DO 56 MTH = 1,ORDER
C                WRITE(6,130) (LRE(MTH,NTH),NTH=1,ORDER)
C   56        CONTINUE
C    CALCULATE THE DETERMINANT OF THE L(M,N) MATRIX:
              CALL DTERM(ORDER,LLE,DETLE,ROWDIM)
              YLE = DETLE
              CALL DTERM(ORDER,LRE,DETRE,ROWDIM)
              YRE = DETRE
              SIGNE = YLE*YRE
              IF(.NOT.(SIGNE.LT.0.0)) GO TO 33
                 XRE = XRESTE
              GO TO 34
    33        CONTINUE
                 XLE = XRESTE
    34        CONTINUE
              EPSLNE = .5*DABS(XLE-XRE)
              ITERE = ITERE + 1
              PRINT,XLE,XRE,ITERE
              GO TO 31
    32        CONTINUE
              IF(.NOT.(ITERE.LT.100)) GO TO 35
                 LERES = .5*(XLE+XRE)
                 XE = DTAN(C2PI*(.5-LERES))
              GO TO 36
    35        CONTINUE
C    WRITE OUTPUT - SEARCH FAILED TO CONVERGE
                 WRITE(6,810)
    36        CONTINUE
C SEARCH FOR ODD MODE RESONANT LENGTH OF INDUCTIVELY COUPLED FINLINE
C CAVITY.
C SEARCH INTERVAL IS XLOMIN TO XROMAX
           XLO = XLOMIN
           XRO = XROMAX
           EPSLNO = .5*DABS(XLO-XRO)
           SIGNO = 0.0
           ITERO = 1
C   CALCULATE INTERIOR POINTS
C   AND TEST FOR END OF SEARCH
    41        IF(.NOT.((EPSLNO.GE.ACCUR).AND.(ITERO.LT.100))) GO TO 42
              XRESTO = .5*(XLO+XRO)
C   CALCULATE L(M,N) MATRIX ELEMENTS;  L(M,N) * ALFA(N) =G(M)
              DO 61 MTH = 1,ORDER
                 DO 62 NTH = MTH,ORDER
                    IPWR = MTH + NTH + 1
                    SIGN = (-1.0)**IPWR
                    CALL IPROD4(MTH,NTH,LPROVL,XLO,YLELO)
```

```
                     LLO(MTH,NTH) =  YLELO*SIGN
                     IF( .NOT.(MTH.NE.NTH)) GO TO 63
                        LLO(NTH,MTH) = LLO(MTH,NTH)
   63                CONTINUE
                     CALL IPROD4(MTH,NTH,LPROVL,XRESTO,YRELO)
                     LRO(MTH,NTH) =  YRELO*SIGN
                     IF(.NOT.(MTH.NE.NTH)) GO TO 64
                        LRO(NTH,MTH) = LRO(MTH,NTH)
   64                CONTINUE
   62             CONTINUE
   61          CONTINUE
C              DO 65 MTH = 1,ORDER
C                 WRITE(6,130) (LLO(MTH,NTH),NTH=1,ORDER)
C  65          CONTINUE
C              DO 66 MTH = 1,ORDER
C                 WRITE(6,130) (LRO(MTH,NTH),NTH=1,ORDER)
C  66          CONTINUE
C   CALCULATE THE DETERMINANT OF THE L(M,N) MATRIX:
               CALL DTERM(ORDER,LLO,DETLO,ROWDIM)
               YLO = DETLO
               CALL DTERM(ORDER,LRO,DETRO,ROWDIM)
               YRO = DETRO
               SIGNO = YLO*YRO
               IF(.NOT.(SIGNO.LT.0.0)) GO TO 43
                  XRO = XRESTO
               GO TO 44
   43          CONTINUE
                  XLO = XRESTO
   44          CONTINUE
               EPSLNO = .5*DABS(XLO-XRO)
               ITERO = ITERO + 1
               PRINT,XLO,XRO,ITERO
               GO TO 41
   42          CONTINUE
               IF(.NOT.(ITERO.LT.100)) GO TO 45
                  LORES = .5*(XLO+XRO)
                  XO = DTAN(C2PI*(.5-LORES))
               GO TO 46
   45          CONTINUE
C   WRITE OUTPUT - SEARCH FAILED TO CONVERGE
                  WRITE(6,820)
   46          CONTINUE
CLCALCULATE EQUIVALENT CIRCUIT REACTANCES XS AND XM FOR INDUCTIVE POST
               XS=.5*(XE+XO)
               XM=.5*(XE-XO)
CLWRITE SOLUTIONS FOR LE/LPR, LO/LPR, XS AND XM
               WRITE(6,800) TOVD,TOVW,LERES,LORES,XS,XM
CSCALCULATE THE SCATTERING PARAMETERS  FOR INDUCTIVE POST
```

```
CS                MAGS11 = DCOS(2.*PI*(LORES-LERES))
CS                RADS11 = 2.*PI*(LERES+LORES) + PI
CS                ANGS11 = (2.*(LERES+LORES)+1.)*180. - 360.
CS                MAGS12 = DSIN(2.*PI*(LORES-LERES))
CS                RADS12 = 2.*PI*(LERES+LORES) - (PI/2.)
CS                ANGS12 = (2.*(LERES+LORES)-.5)*180. - 180.
CSWRITE SOLUTIONS FOR S11, THETA11, S12, AND THETA12
CS                WRITE(6,800) TOVD,TOVW,MAGS11,ANGS11,MAGS12,ANGS12
   28         CONTINUE
   11       CONTINUE
   12 CONTINUE
   13 STOP
  100 FORMAT(8F10.8)
  101 FORMAT(7F10.8)
  102 FORMAT(2F10.8)
  110 FORMAT(' THIS IS THE FINSTRP PROGRAM TO DETERMINE THE ODD AND'/
     *1X,' EVEN MODE RESONANT LENGTHS OF TWO UNILATERALLY COUPLED'/
     *1X,' FIN-LINE RESONANT CAVITIES, THE CORRESPONDING EQUIVALENT'/
     *1X,' CIRCUIT REACTANCES, AND THE SCATTERING PARAMETERS OF THE'/
     *1X,' INCUCTIVE STRIP USING A BASIS SET OF SIN AND COS'/
     *1X,' FUNCTIONS'/)
  120 FORMAT(' THE DIMENSION OF THE MATRIX IS = ',I4,4X,'EPSLN=E-4'//)
C 130 FORMAT(1X,2F10.7/)
  200 FORMAT(' FINLINE PARAMETERS FOR DATA SET#2 ARE'/)
  300 FORMAT(' EPSR1 = ',F4.1,5X,'EPSR2 = ',F4.1,5X,'EPSR3 = ',
     1F4.1)
  400 FORMAT(' H1/D = ',F4.1,6X,'H2/D = ',F4.1,6X,'B/D = ',F4.1)
  489 FORMAT('    W/B    ','    W/D    ','    D/L    ','    LPR/L    ')
CL499 FORMAT FOR T/D,T/W, LE/LPR, LO/LPR, XS AND XM
  499 FORMAT(/10X,'   TOVD    ','   TOVW    ','  LE/LPR   ','  LO/LPR   '
     *,'    XS     ','    XM     ')
CS499 FORMAT FOR T/D, T/W, MAGS11, ANGS11, MAGS12, AND ANGS12
CS499 FORMAT(/10X,'   TOVD    ','   TOVW    ','  MAGS11   ','  ANGS11   '
CS   *,'  MAGS12   ','  ANGS12   ')
C 500 FORMAT FOR W/B, W/D AND D/L
  500 FORMAT(/F7.3,3X,F7.3,3X,F7.4)
C 600 FORMAT FOR LPR/L
  600 FORMAT(30X,F7.3)
  610 FORMAT('+',30X,'    MSG2    '/)
CL800 FORMAT FOR T/D,T/W, LE/LPR, LO/LPR, XS AND XM
  800 FORMAT(10X,F7.3,3X,F7.3,3X,F7.3,3X,F7.3,3X,F7.3,3X,F7.3)
CS800 FORMAT FOR T/D, T/W, MAGS11, ANGS11, MAGS12, AND ANGS12
CS800 FORMAT(10X,F7.3,3X,F7.3,3X,F7.3,3X,F7.3,3X,F7.3,3X,F7.3)
C 805 FORMAT('+',40X,F7.3,3X,'    MSG3    '/)
  810 FORMAT('+',10X,F7.3,3X,'    MSG4    ','    MSG4    '/)
  820 FORMAT('+',30X,'    MSG5    '/)
C ERROR MESSAGES
C MSG2: SEARCH FOR ZERO OF IPROD1 FAILED TO CONVERGE
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C MSG3: SEARCH FOR ZERO OF IPROD2 FAILED TO CONVERGE
C MSG4: SEARCH FOR ZERO OF IPROD3 FAILED TO CCNVERGE
C MSG5: SEARCH FOR ZERO OF IPROD4 FAILED TO CONVERGE
      END
C SUBR*****************************************************************
$EJECT
      SUBROUTINE IPROD1(LPROVL,SUM1)
C**********************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE INNER PRODUCT OF   *
C THE G11 DYADIC GREEN'S FUNCTION AND THE SQUARE CF THE FOURIER TRANS*
C -FORM OF THE X-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD           *
C*****VARIABLE DEFINITIONS*********************************************
C EXXSQ = SQUARE OF THE FOURIER TRANSFORM OF THE X-COMPONENT OF THE   *
C         X-DIRECTED ELECTRIC FIELD                                   *
C G11 = THE G11 DYADIC GREEN'S FUNCTION                               *
C MMAX = THE LIMIT OF SUMMATION OVER ALFD                             *
C SUM1 = THE RETURNED VALUE OF THE INNER PRODUCT                      *
C*****VARIABLE DECLARATION*********************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      REAL*8 LPROVL
      COMMON/C1/EPSR1,EPSR2,EPSR3,H1OVD,H2OVD,BOVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      COMMON/C5/XCONST,ZCONST
      BETAD = C2PI*DOVL/LPROVL
      BOVW = 1./WOVB
C   SPECIFY LIMIT FOR SUMMATION ON N
      IF(WOVB.EQ.1.) MMAX=1
      IF(WOVB.LT.1.) MMAX=IDINT((XCONST*BOVW)/C2PI)
      IF(MMAX.GT.40) MMAX=40
      SUM1 = 0.0
      DO 11 M=1,MMAX
         N=M-1
         ALFD=N*C2PI/BOVD
         CALL GRN11(ALFD,BETAD,DOVL,G11)
         CALL EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
         SUM1=SUM1+2.*G11*EXXSQ
         IF(N.EQ.0) SUM1=G11
   11 CONTINUE
      RETURN
      END
C SUBR*****************************************************************
$EJECT
      SUBROUTINE IPROD3(MTH,NTH,LPROVL,LERES,SUM3)
C**********************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE INNER PRODUCT OF   *
C THE G11 DYADIC GREEN'S FUNCTION, THE FOURIER TRANSFORM OF THE MTH   *
C Z-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD, THE FOURIER TRANSFORM*
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C OF THE NTH Z-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD, AND THE    *
C MTH AND NTH SPATIAL SHIFT FUNCTIONS FOR THE Z-COMPONENT FUNCTIONS    *
C WITH AN ESTIMATE OF THE EVEN MODE RESONANT LENGTH AS AN ARGUMENT.    *
C IF M OR N IS EVEN, THE FOURIER TRANSFORM OF THE Z-COMPONENT IS A     *
C SIN FUNCTION WITH AN INDEX VALUE OF MTH/2 OR NTH/2.                  *
C IF M OR N IS ODD, THE FOURIER TRANSFORM OF THE Z-COMPONENT IS A      *
C COS FUNCTION WITH AN INDEX VALUE OF (MTH+1)/C OR (NTH+1)/2.          *
C*****VARIABLE DEFINITIONS**********************************************
C EXXSQ = SQUARE OF THE FOURIER TRANSFORM OF THE X-COMPONENT OF THE    *
C         X-DIRECTED ELECTRIC FIELD                                    *
C G11 = THE G11 DYADIC GREEN'S FUNCTION                                *
C EXZM = THE FOURIER TRANSFORM OF THE MTH Z-COMPONENT OF THE           *
C         X-DIRECTED ELECTRIC FIELD                                    *
C EXZN = THE FOURIER TRANSFORM OF THE NTH Z-COMPONENT OF THE           *
C         X-DIRECTED ELECTRIC FIELD                                    *
C EXZCM = THE FOURIER TRANSFORM OF THE MTH Z-COMPONENT OF THE          *
C         ELECTRIC FIELD WHERE M IS ODD AND THE SPATIAL FUNCTION IS A*
C         COSINE FUNCTION.                                             *
C EXZCN = THE FOURIER TRANSFORM OF THE NTH Z-COMPONENT OF THE          *
C         ELECTRIC FIELD WHERE N IS ODD AND THE SPATIAL FUNCTION IS A*
C         COSINE FUNCTION.                                             *
C EXZE3M = THE FOURIER TRANSFORM OF THE SPATIAL SHIFT OF THE MTH Z-    *
C          COMPONENT OF THE ELECTRIC FIELD FOR THE EVEN MODE WHERE     *
C          M IS ODD AND THE SHIFT IS A COSINE FUNCTION.                *
C EXZE3N = THE FOURIER TRANSFORM OF THE SPATIAL SHIFT OF THE NTH Z-    *
C          COMPONENT OF THE ELECTRIC FIELD FOR THE EVEN MODE WHERE     *
C          N IS ODD AND THE SHIFT IS A COSINE FUNCTION.                *
C EXZE5M = THE FOURIER TRANSFORM OF THE SPATIAL SHIFT OF THE MTH Z-    *
C          COMPONENT OF THE ELECTRIC FIELD FOR THE EVEN MODE WHERE     *
C          M IS EVEN AND THE SHIFT IS A SINE FUNCTION.                 *
C EXZE5N = THE FOURIER TRANSFORM OF THE SPATIAL SHIFT OF THE NTH Z-    *
C          COMPONENT OF THE ELECTRIC FIELD FOR THE EVEN MODE WHERE     *
C          N IS EVEN AND THE SHIFT IS A SINE FUNCTION.                 *
C EXZSM = THE FOURIER TRANSFORM OF THE MTH Z-COMPONENT OF THE          *
C         ELECTRIC FIELD WHERE M IS EVEN AND THE SPATIAL FUNCTION IS *
C         A SINE FUNCTION.                                             *
C EXZSN = THE FOURIER TRANSFORM OF THE NTH Z-COMPONENT OF THE          *
C         ELECTRIC FIELD WHERE N IS EVEN AND THE SPATIAL FUNCTION IS *
C         A SINE FUNCTION.                                             *
C LEOVD = EVEN MODE RESONANT LENGTH/D                                  *
C LERES = EST OF THE EVEN MODE RESONANCE LENGTH/GUIDE WAVELENGTH       *
C MTH = ROW INDEX                                                      *
C NTH = COLUMN INDEX                                                   *
C NMAX = THE LIMIT OF SUMMATION OVER ALFD                              *
C KMAX = THE LIMIT OF SUMMATION OVER ZETAD                             *
C SUM3 = THE RETURNED VALUE OF THE INNER PRODUCT                       *
C SHIFTM = THE FOURIER TRANSFORM OF THE SPATIAL SHIFT OF THE MTH Z-    *
C          COMPONENT OF THE ELECTRIC FIELD                             *
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C SHIFTN = THE FOURIER TRANSFORM OF THE SPATIAL SHIFT OF THE NTH Z-   *
C          COMPONENT OF THE ELECTRIC FIELD                            *
C TOVLE = SEPTUM THICKNESS/EVEN MODE RESONANT LENGTH                  *
C*****VARIABLE DECLARATION********************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      INTEGER REMAIM,REMAIN
      REAL*8 LPROVL,LERES,LEOVD
      COMMON/C1/EPSR1,EPSR2,EPSR3,H1OVD,H2OVD,BOVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      COMMON/C4/TOVD
      COMMON/C5/XCONST,ZCONST
      BOVW = 1./WOVB
      LEOVD=LERES*LPROVL/DOVL
      TOVLE=TOVD/LEOVD
C SPECIFY LIMIT FOR SUMMATION ON N
      IF(WOVB.EQ.1.) NMAX=0
      IF(WOVB.LT.1.) NMAX=IDINT((XCONST*BOVW)/C2PI)
      IF(NMAX.GT.40) NMAX=40
C SPECIFY LIMIT FOR SUMMATION ON K
      RMN = AMAXO(MTH,NTH)
      KMAX=IDINT(1.5 + (2.+TOVLE)*DSQRT(ZCONST+((2.*RMN-1.)/2.)**2))
      PRINT,NMAX,KMAX
      SUM3 = 0.0
      N=0
      DO 10 K = 1,KMAX
         RN = DFLOAT(N)
         RK = DFLOAT(K)
         ALFD = RN*C2PI/BOVD
         ZETAD = ((RK-.5)*C2PI)/(2.*LEOVD+TOVD)
         BETAD = ZETAD
         CALL GRN11(ALFD,BETAD,DOVL,G11)
         CALL EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
C DETERMINE IF MTH IS EVEN OR ODD
         REMAIM = MOD(MTH,2)
         IF(.NOT.(REMAIM.EQ.0)) GO TO 15
            MM = MTH/2
            CALL EXZSIN(MM,ZETAD,LERES,LPROVL,EXZSM)
            EXZM = EXZSM
            CALL EXZSH5(ZETAD,LERES,LPROVL,TOVD,EXZE5M)
            SHIFTM = EXZE5M
         GO TO 16
   15    CONTINUE
            MM = (MTH+1)/2
            CALL EXZCOS(MM,ZETAD,LERES,LPROVL,EXZCM)
            EXZM = EXZCM
            CALL EXZSH3(ZETAD,LERES,LPROVL,TOVD,EXZE3M)
            SHIFTM = EXZE3M
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```
   16     CONTINUE
C DETERMINE IF NTH IS EVEN OR ODD
          REMAIN = MOD(NTH,2)
          IF(.NOT.(REMAIN.EQ.0)) GO TO 17
             NN = NTH/2
             CALL EXZSIN(NN,ZETAD,LERES,LPROVL,EXZSN)
             EXZN = EXZSN
             CALL EXZSH5(ZETAD,LERES,LPROVL,TOVD,EXZE5N)
             SHIFTN = EXZE5N
          GO TC 18
   17     CONTINUE
             NN = (NTH+1)/2
             CALL EXZCOS(NN,ZETAD,LERES,LPROVL,EXZCN)
             EXZN = EXZCN
             CALL EXZSH3(ZETAD,LERES,LPROVL,TOVD,EXZE3N)
             SHIFTN = EXZE3N
   18     CONTINUE
          SUM3=SUM3+2.*G11*EXXSQ*EXZM*EXZN*SHIFTM*SHIFTN
   10  CONTINUE
       IF(NMAX.EQ.0) GO TO 14
       DO 13 N = 1,NMAX
          DO 12 K = 1,KMAX
             RN = DFLOAT(N)
             RK = DFLOAT(K)
             ALFD = RN*C2PI/BOVD
             ZETAD = ((RK-.5)*C2PI)/(2.*LEOVD+TOVD)
             BETAD = ZETAD
             CALL GRN11(ALFD,BETAD,DOVL,G11)
             CALL EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
C DETERMINE IF MTH IS EVEN OR ODD
             REMAIM = MOD(MTH,2)
             IF(.NOT.(REMAIM.EQ.0)) GO TO 25
                MM = MTH/2
                CALL EXZSIN(MM,ZETAD,LERES,LPROVL,EXZSM)
                EXZM = EXZSM
                CALL EXZSH5(ZETAD,LERES,LPROVL,TOVC,EXZE5M)
                SHIFTM = EXZE5M
             GC TO 26
   25        CONTINUE
                MM = (MTH+1)/2
                CALL EXZCOS(MM,ZETAD,LERES,LPRCVL,EXZCM)
                EXZM = EXZCM
                CALL EXZSH3(ZETAD,LERES,LPROVL,TOVC,EXZE3M)
                SHIFTM = EXZE3M
   26        CCNTINUE
C DETERMINE IF NTH IS EVEN OR ODD
             REMAIN = MOD(NTH,2)
             IF(.NOT.(REMAIN.EQ.0)) GO TO 27
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```
          NN =  NTH/2
          CALL  EXZSIN(NN,ZETAD,LERES,LPROVL,EXZSN)
          EXZN = EXZSN
          CALL  EXZSH5(ZETAD,LERES,LPROVL,TOVD,EXZE5N)
          SHIFTN = EXZE5N
        GO TO 28
   27   CONTINUE
          NN = (NTH+1)/2
          CALL  EXZCOS(NN,ZETAD,LERES,LPROVL,EXZCN)
          EXZN = EXZCN
          CALL  EXZSH3(ZETAD,LERES,LPROVL,TOVC,EXZE3N)
          SHIFTN = EXZE3N
   28   CONTINUE
        SUM3=SUM3+4.*G11*EXXSQ*EXZM*EXZN*SHIFTM*SHIFTN
   12  CONTINUE
   13 CONTINUE
   14 CONTINUE
      RETURN
      END
C SUBR***************************************************************
$EJECT
      SUBROUTINE IPROD4(MTH,NTH,LPROVL,LORES,SUM4)
C*******************************************************************
C      THIS SUBROUTINE IS DESIGNED TO CALCULATE THE INNER PRODUCT OF   *
C THE G11 DYADIC GREEN'S FUNCTION, THE FOURIER TRANSFORM OF THE MTH   *
C Z-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD, THE FOURIER TRANSFORM*
C OF THE NTH Z-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD, AND THE    *
C MTH AND NTH SPATIAL SHIFT FUNCTIONS FOR THE Z-COMPONENT FUNCTIONS   *
C WITH AN ESTIMATE OF THE ODD  MODE RESONANT LENGTH AS AN ARGUMENT.   *
C IF M OR N IS EVEN, THE FOURIER TRANSFORM OF THE Z-COMPONENT IS A    *
C SIN FUNCTION WITH AN INDEX VALUE OF MTH/2 OR NTH/2.                 *
C IF M OR N IS ODD, THE FOURIER TRANSFORM OF THE Z-COMPONENT IS A     *
C COS FUNCTION WITH AN INDEX VALUE OF (MTH+1)/C OR (NTH+1)/2.         *
C*****VARIABLE DEFINITIONS*******************************************
C EXXSQ = SQUARE OF THE FOURIER TRANSFORM OF THE X-COMPONENT OF THE   *
C         X-DIRECTED ELECTRIC FIELD                                   *
C G11 = THE G11 DYADIC GREEN'S FUNCTION                               *
C EXZM = THE FOURIER TRANSFORM OF THE MTH Z-COMPONENT OF THE          *
C         X-DIRECTED ELECTRIC FIELD                                   *
C EXZN = THE FOURIER TRANSFORM OF THE NTH Z-COMPONENT OF THE          *
C         X-DIRECTED ELECTRIC FIELD                                   *
C EXZCM = THE FOURIER TRANSFORM OF THE MTH Z-COMPONENT OF THE         *
C         ELECTRIC FIELD WHERE M IS ODD AND THE SPATIAL FUNCTION IS A*
C         COSINE FUNCTION.                                            *
C EXZCN = THE FOURIER TRANSFORM OF THE NTH Z-COMPONENT OF THE         *
C         ELECTRIC FIELD WHERE N IS ODD AND THE SPATIAL FUNCTION IS A*
C         COSINE FUNCTION.                                            *
C EXZO4M = THE FOURIER TRANSFORM OF THE SPATIAL SHIFT OF THE MTH Z-   *
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C            COMPONENT OF THE ELECTRIC FIELD FOR THE ODD MODE WHERE      *
C            M IS ODD AND THE SHIFT IS A SINE FUNCTION.                  *
C  EXZO4N =  THE FOURIER TRANSFORM OF THE SPATIAL SHIFT OF THE NTH Z-    *
C            COMPONENT OF THE ELECTRIC FIELD FOR THE ODD MODE WHERE      *
C            N IS ODD AND THE SHIFT IS A SINE FUNCTION.                  *
C  EXZO6M =  THE FOURIER TRANSFORM OF THE SPATIAL SHIFT OF THE MTH Z-    *
C            COMPONENT OF THE ELECTRIC FIELD FOR THE ODD MODE WHERE      *
C            M IS EVEN AND THE SHIFT IS A COSINE FUNCTION.               *
C  EXZO6N =  THE FOURIER TRANSFORM OF THE SPATIAL SHIFT OF THE NTH Z-    *
C            COMPONENT OF THE ELECTRIC FIELD FOR THE ODD MODE WHERE      *
C            N IS EVEN AND THE SHIFT IS A COSINE FUNCTION.               *
C  EXZSM = THE FOURIER TRANSFORM OF THE MTH Z-COMPONENT OF THE           *
C          ELECTRIC FIELD WHERE M IS EVEN AND THE SPATIAL FUNCTION IS    *
C          A SINE FUNCTION.                                              *
C  EXZSN = THE FOURIER TRANSFORM OF THE NTH Z-COMPONENT OF THE           *
C          ELECTRIC FIELD WHERE N IS EVEN AND THE SPATIAL FUNCTION IS    *
C          A SINE FUNCTION.                                              *
C  LOOVD = ODD MODE RESONANT LENGTH/D                                    *
C  LORES = EST OF THE ODD MODE RESONANCE LENGTH/GUIDE WAVELENGTH         *
C  MTH = ROW INDEX                                                       *
C  NTH = COLUMN INDEX                                                    *
C  NMAX = THE LIMIT OF SUMMATION OVER ALFD                               *
C  KMAX = THE LIMIT OF SUMMATION OVER ZETAD                              *
C  SUM4 = THE RETURNED VALUE OF THE INNER PRODUCT                        *
C  SHIFTM = THE FOURIER TRANSFORM OF THE SPATIAL SHIFT OF THE MTH Z-     *
C           COMPONENT OF THE ELECTRIC FIELD                              *
C  SHIFTN = THE FOURIER TRANSFORM OF THE SPATIAL SHIFT OF THE NTH Z-     *
C           COMPONENT OF THE ELECTRIC FIELD                              *
C  TOVLO = SEPTUM THICKNESS/ODD MODE RESONANT LENGTH                     *
C*****VARIABLE DECLARATION***********************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      REAL*8 LPROVL,LORES,LOOVD
      COMMON/C1/EPSR1,EPSR2,EPSR3,H1OVD,H2OVD,BOVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      COMMON/C4/TOVD
      COMMON/C5/XCONST,ZCONST
      BOVW = 1./WOVB
      LOOVD=LORES*LPROVL/DOVL
      TOVLO=TOVD/LOOVD
C SPECIFY LIMIT FOR SUMMATION ON N
      IF(WOVB.EQ.1.) NMAX=0
      IF(WOVB.LT.1.) NMAX=IDINT((XCONST*BOVW)/C2PI)
      IF(NMAX.GT.40) NMAX=40
C SPECIFY LIMIT FOR SUMMATION ON K
      RMN = AMAXO(MTH,NTH)
      KMAX=IDINT((2.+TOVLO)*DSQRT(ZCONST+((2.*RMN-1.)/2.)**2))
      PRINT,NMAX,KMAX
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      SUM4 = 0.0
      N=0
      K=0
      RN = DFLOAT(N)
      RK = DFLOAT(K)
      ALFD=RN*C2PI/BOVD
      ZETAD = (RK*C2PI)/(2.*LDOVD+TOVD)
      BETAD = ZETAD
      CALL GRN11(ALFD,BETAD,DOVL,G11)
      CALL EXSC1(ALFD,WOVB,BOVD,EXXSQ)
C DETERMINE IF MTH IS EVEN OR ODD
      REMAIM = MOD(MTH,2)
      IF(.NOT.(REMAIM.EQ.0)) GO TO 5
         MM = MTH/2
         CALL EXZSIN(MM,ZETAC,LORES,LPROVL,EXZSM)
         EXZM = EXZSM
         CALL EXZSH6(ZETAD,LORES,LPROVL,TOVD,EXZO6M)
         SHIFTM = EXZO6M
      GO TO 6
    5 CONTINUE
         MM = (MTH+1)/2
         CALL EXZCOS(MM,ZETAD,LORES,LPROVL,EXZCM)
         EXZM = EXZCM
         CALL EXZSH4(ZETAD,LORES,LPROVL,TOVD,EXZO4M)
         SHIFTM = EXZO4M
    6 CONTINUE
C DETERMINE IF NTH IS EVEN OR ODD
      REMAIN = MOD(NTH,2)
      IF(.NOT.(REMAIN.EQ.0)) GO TO 7
         NN = NTH/2
         CALL EXZSIN(NN,ZETAC,LORES,LPROVL,EXZSN)
         EXZN = EXZSN
         CALL EXZSH6(ZETAD,LORES,LPROVL,TOVD,EXZO6N)
         SHIFTN = EXZO6N
      GO TO 8
    7 CONTINUE
         NN = (NTH+1)/2
         CALL EXZCOS(NN,ZETAD,LORES,LPROVL,EXZCN)
         EXZN = EXZCN
         CALL EXZSH4(ZETAD,LORES,LPROVL,TOVD,EXZO4N)
         SHIFTN = EXZO4N
    8 CONTINUE
      SUM4=G11*EXXSQ*EXZM*EXZN*SHIFTM*SHIFTN
      N=0
      DO 10 K = 1,KMAX
         RN = DFLOAT(N)
         RK = DFLOAT(K)
         ALFD = RN*C2PI/BOVD
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         ZETAD = (RK*C2PI)/(2.*LOOVD+TOVD)
         BETAD = ZETAD
         CALL GRN11(ALFD,BETAD,DOVL,G11)
         CALL EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
C DETERMINE IF MTH IS EVEN OR ODD
         REMAIM = MOD(MTH,2)
         IF(.NOT.(REMAIM.EQ.0)) GO TO 15
            MM = MTH/2
            CALL EXZSIN(MM,ZETAD,LORES,LPROVL,EXZSM)
            EXZM = EXZSM
            CALL EXZSH6(ZETAD,LORES,LPROVL,TOVD,EXZ06M)
            SHIFTM = EXZ06M
         GO TO 16
   15    CONTINUE
            MM = (MTH+1)/2
            CALL EXZCOS(MM,ZETAD,LORES,LPROVL,EXZCM)
            EXZM = EXZCM
            CALL EXZSH4(ZETAD,LORES,LPROVL,TOVD,EXZ04M)
            SHIFTM = EXZ04M
   16    CONTINUE
C DETERMINE IF NTH IS EVEN OR ODD
         REMAIN = MOD(NTH,2)
         IF(.NOT.(REMAIN.EQ.0)) GO TO 17
            NN = NTH/2
            CALL EXZSIN(NN,ZETAD,LORES,LPROVL,EXZSN)
            EXZN = EXZSN
            CALL EXZSH6(ZETAD,LORES,LPROVL,TOVD,EXZ06N)
            SHIFTN = EXZ06N
         GO TO 18
   17    CONTINUE
            NN = (NTH+1)/2
            CALL EXZCOS(NN,ZETAD,LORES,LPROVL,EXZCN)
            EXZN = EXZCN
            CALL EXZSH4(ZETAD,LORES,LPROVL,TOVD,EXZ04N)
            SHIFTN = EXZ04N
   18    CONTINUE
         SUM4=SUM4+2.*G11*EXXSQ*EXZM*EXZN*SHIFTM*SHIFTN
   10 CONTINUE
      IF(NMAX.EQ.0) GO TO 14
      K=0
      DO 11 N = 1,NMAX
         RN = DFLOAT(N)
         RK = DFLOAT(K)
         ALFD = RN*C2PI/BOVD
         ZETAD = (RK*C2PI)/(2.*LOOVD+TOVD)
         BETAD = ZETAD
         CALL GRN11(ALFD,BETAD,DOVL,G11)
         CALL EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
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C DETERMINE  IF MTH IS EVEN OR ODD
          REMAIM = MOD(MTH,2)
          IF( .NOT. (REMAIM.EQ.0)) GO TO 25
             MM = MTH/2
             CALL EXZSIN(MM,ZETAD,LORES,LPROVL,EXZSM)
             EXZM = EXZSM
             CALL EXZSH6(ZETAD,LORES,LPROVL,TOVD,EXZO6M)
             SHIFTM = EXZO6M
          GO TO 26
   25     CONTINUE
             MM = (MTH+1)/2
             CALL EXZCOS(MM,ZETAD,LORES,LPROVL,EXZCM)
             EXZM = EXZCM
             CALL EXZSH4(ZETAD,LORES,LPROVL,TOVD,EXZO4M)
             SHIFTM = EXZO4M
   26     CONTINUE
C DETERMINE  IF NTH IS EVEN OR ODD
          REMAIN = MOD(NTH,2)
          IF( .NOT. (REMAIN.EQ.0)) GO TO 27
             NN = NTH/2
             CALL EXZSIN(NN,ZETAD,LORES,LPROVL,EXZSN)
             EXZN = EXZSN
             CALL EXZSH6(ZETAD,LORES,LPROVL,TOVD,EXZO6N)
             SHIFTN = EXZO6N
          GO TO 28
   27     CONTINUE
             NN = (NTH+1)/2
             CALL EXZCOS(NN,ZETAD,LORES,LPROVL,EXZCN)
             EXZN = EXZCN
             CALL EXZSH4(ZETAD,LORES,LPROVL,TOVD,EXZO4N)
             SHIFTN = EXZO4N
   28     CONTINUE
          SUM4=SUM4+2.*G11*EXXSQ*EXZM*EXZN*SHIFTM*SHIFTN
   11  CONTINUE
       DO 13 N = 1,NMAX
          DO 12 K = 1,KMAX
             RN = DFLOAT(N)
             RK = DFLOAT(K)
             ALFD = RN*C2PI/BOVD
             ZETAD = (RK*C2PI)/(2.*LOOVD+TOVD)
             BETAD = ZETAD
             CALL GRN11(ALFD,BETAD,DOVL,G11)
             CALL EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
C DETERMINE  IF MTH IS EVEN OR ODD
             REMAIM = MOD(MTH,2)
             IF(.NOT.(REMAIM.EQ.0)) GO TO 35
                MM = MTH/2
                CALL EXZSIN(MM,ZETAD,LORES,LPROVL,EXZSM)
```

```
          EXZM = EXZSM
          CALL EXZSH6(ZETAD,LORES,LPROVL,TOVC,EXZ06M)
          SHIFTM = EXZ06M
        GC TO 36
 35     CCNTINUE
          MM = (MTH+1)/2
          CALL EXZCOS(MM,ZETAD,LORES,LPROVL,EXZCM)
          EXZM = EXZCM
          CALL EXZSH4(ZETAD,LORES,LPROVL,TOVD,EXZ04M)
          SHIFTM = EXZ04M
 36     CCNTINUE
C DETERMINE IF NTH IS EVEN OR ODD
        REMAIN = MOD(NTH,2)
        IF(.NOT.(REMAIN.EQ.0)) GO TO 37
          NN = NTH/2
          CALL EXZSIN(NN,ZETAD,LORES,LPRCVL,EXZSN)
          EXZN = EXZSN
          CALL EXZSH6(ZETAD,LORES,LPROVL,TOVC,EXZ06N)
          SHIFTN = EXZ06N
        GC TO 38
 37     CCNTINUE
          NN = (NTH+1)/2
          CALL EXZCOS(NN,ZETAD,LORES,LPROVL,EXZCN)
          EXZN = EXZCN
          CALL EXZSH4(ZETAD,LORES,LPROVL,TOVD,EXZ04N)
          SHIFTN = EXZ04N
 38     CCNTINUE
        SUM4=SJM4+4.*G11*EXXSQ*EXZM*EXZN*SHIFTM*SHIFTN
 12     CONTINUE
 13   CONTINUE
 14   CONTINUE
      RETURN
      END
C SUBR***********************************************************************
$EJECT
      SUBROUTINE GRN11(ALFD,BETAD,DOVL,G11)
C****************************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE G11 DYADIC GREEN'S*
C FUNCTION                                                                   *
C*****VARIABLE DEFINITIONS**************************************************
C G11 = THE G11 DYADIC GREEN'S FUNCTION, RETURNED VALUE                     *
C G1DSQ                                                                      *
C G2DSQ                                                                      *
C G3DSQ                                                                      *
C KC1DSQ                                                                     *
C KC2DSQ                                                                     *
C KC3DSQ                                                                     *
C WEPS1D                                                                     *
```

```
C WEPS2D                                                              *
C WEPS3D                                                              *
C WMUD                                                                *
C*****VARIABLE DECLARATION*********************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      REAL*8 KC1DSQ,KC2DSQ,KC3DSQ,LPROVL,LOVLPR
      COMMON/C1/EPSR1,EPSR2,EPSR3,H1OVD,H2OVD,BOVD
      COMMON/C2/C2PI,C2PISQ,PI
C   CALCULATE VARIABLES DEPENDENT ON FREQUENCY
      WMUD=60.*C2PISQ*DOVL
      WEPS1D=EPSR1*DOVL/60.
      WEPS2D=EPSR2*DOVL/60.
      WEPS3D=EPSR3*DOVL/60.
C   CALCULATE VARIABLES DEPENDENT ON FRRQUENCY AND BETAD
      BETDSQ=BETAD**2
      KC1DSQ=C2PISQ*EPSR1*DOVL**2-BETDSQ
      KC2DSQ=C2PISQ*EPSR2*DOVL**2-BETDSQ
      KC3DSQ=C2PISQ*EPSR3*DOVL**2-BETDSQ
C   CALCULATE VARIABLES DEPENDENT ON FREQUENCY, BETAD AND ALFD
      ALFDSQ=ALFD**2
      G1DSQ=ALFDSQ-KC1DSQ
      G2DSQ=ALFDSQ-KC2DSQ
      G3DSQ=ALFDSQ-KC3DSQ
      CALL TFN(G1DSQ,H1OVD,TFN1)
      CALL TFN(G2DSQ,1.0D0,TFN2)
      CALL TFN(G3DSQ,H2OVD,TFN3)
      D11=-KC2DSQ*(1.+(WEPS3D*G3DSQ*KC2DSQ*TFN2)/(WEPS2D*G2DSQ*KC3DSQ*
     1TFN3))
      D12=((ALFD*BETAD)/(WEPS2D*G2DSQ))*((KC2DSQ/KC1DSQ)-1.)*KC2DSQ
      D21=-ALFD*BETAD*((KC2DSQ/KC1DSQ)+((WEPS3D*KC2DSQ*G3DSQ*TFN2)/
     1(WEPS2D*KC3DSQ*G2DSQ*TFN3)))
      D22=WMUD*(1.+((KC2DSQ*TFN3)/(KC3DSQ*TFN2))+((ALFDSQ*BETDSQ)/(G2DSQ
     1*WMUD*WEPS2D))*((KC2DSQ/KC3DSQ)-1.))
      DET=D11*D22-D21*D12
C   CALCULATE G11
      G11A=-KC1DSQ/(WMUD*TFN1)
      G11B=-(KC2DSQ*D12*ALFD*BETAD*KC2DSQ*TFN2)/(DET*WMUD*G2DSQ*KC3DSQ)
      G11C=(KC2DSQ*D12*ALFD*BETAD*TFN2)/(DET*WMUD*G2DSQ)
      G11D=-(KC2DSQ*D11*KC2DSQ*TFN3)/(DET*KC3DSQ*G2DSQ)
      G11E=-(KC2DSQ*D11)/(DET*TFN2)
      G11=G11A+G11B+G11C+G11D+G11E
      RETURN
      END
C SUBR*****************************************************************
$EJECT
      SUBROUTINE TFN(GIDSQ,HIOVD,TFNI)
C*********************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE TANGENT AND THE    *
```

```
C HYPERBOLIC TANGENT FUNCTIONS FOR THE GRN11 SUBRCUTINE.                *
C*****VARIABLE DEFINITIONS**********************************************
C GID                                                                   *
C GIDSQ                                                                 *
C HIOVD                                                                 *
C TFNI = THE RETURNED VALUE                                             *
C*****VARIABLE DECLARATION**********************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      GID=DSQRT(DABS(GIDSQ))
      ARG=GICSQ*HIOVD**2
      IF(ARG.LE.0.) GO TO 1
      IF((ARG.GT.0.).AND.(ARG.LT.100.)) GO TO 2
      IF(ARG.GE.100.) GO TO 3
    1 TFNI=-GID*DTAN(GID*HIOVD)
      GO TO 4
    2 TFNI=GIC*DTANH(GID*HIOVD)
      GO TO 4
    3 TFNI=GID
    4 RETURN
      END
C SUBR******************************************************************
$EJECT
      SUBROUTINE EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
C***********************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE FOURIER TRANSFORM *
C OF THE X-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD AND SQUARE IT.  *
C*****VARIABLE DEFINITIONS**********************************************
C EXXSQ = SQUARE OF THE FOURIER TRANSFORM OF THE X-COMPONENT OF THE    *
C         X-DIRECTED ELECTRIC FIELD, THE RETURNED VALUE                *
C*****VARIABLE DECLARATION**********************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      COMMON/C2/C2PI,C2PISQ,PI
      WOVD=WOVB*BOVD
C     CALCULATE EXXSQ, THE X TRANSFORM OF THE E FIELD BETWEEN THE FINS
      IF(ALFD.EQ.0.) EXXSQ=1.0
      IF(ALFD.GT.0.) EXXSQ=(DSIN(.5*ALFD*WOVD)/(.5*ALFD*WOVD))**2
      RETURN
      END
C SUBR******************************************************************
$EJECT
      SUBROUTINE EXZCOS(MN,ZETAD,LRES,LPROVL,EXZCMN)
C***********************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE FOURIER TRANSFORM *
C OF THE Z-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD WHERE THE        *
C SPATIAL FUNCTION IS A COSINE FUNCTION.                               *
C*****VARIABLE DEFINITIONS**********************************************
C EXZCMN = THE FOURIER TRANSFORM OF THE MNTH COS Z-COMPONENT OF THE    *
C         X-DIRECTED ELECTRIC FIELD, THE RETURNED VALUE.               *
```

```
C MN = ROW OR COLUMN INDEX                                                  *
C*****VARIABLE DECLARATION**************************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      REAL*8 LRES,LPROVL,LOVD,NUMER,DENOM,DIM
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      LOVD=LRES*LPROVL/DOVL
      THETA=ZETAD*.5*LOVD
      DIM = DFLOAT(MN)
      RESID = DABS(THETA - (((2.*DIM-1.)*PI)/2.))
      SIGN = (-1.0)**MN
      NUMER = DCOS(THETA)
      DENOM = ((THETA)**2) - (((((2.*DIM)-1.)*PI)/2.)**2)
      IF(.NOT.(RESID.LT.1.0D-7)) GO TO 35
         EXZCMN = 1./PI
      GO TO 36
   35 CONTINUE
         EXZCMN = SIGN*.5*((2.*DIM)-1.0)*(NUMER/DENOM)
   36 CONTINUE
      RETURN
      END
C SUBR**********************************************************************
$EJECT
      SUBROUTINE EXZSIN(MN,ZETAD,LRES,LPROVL,EXZSMN)
C***************************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE FOURIER TRANSFORM *
C OF THE Z-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD WHERE THE      *
C SPATIAL FUNCTION IS A SINE FUNCTION.                               *
C*****VARIABLE DEFINITIONS**************************************************
C EXZSMN = THE FOURIER TRANSFORM OF THE MNTH SIN Z-COMPONENT OF THE  *
C          X-DIRECTED ELECTRIC FIELD, THE RETURNED VALUE.            *
C MN = ROW OR COLUMN INDEX                                           *
C*****VARIABLE DECLARATION**************************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      REAL*8 LRES,LPROVL,LOVD,NUMER,DENOM,DIM
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      LOVD=LRES*LPROVL/DOVL
      THETA=ZETAD*.5*LOVD
      DIM = DFLOAT(MN)
      RESID = DABS(THETA - (DIM*PI))
      SIGN = (-1.0)**MN
      NUMER = DSIN(THETA)
      DENOM = ((THETA)**2) - ((DIM*PI)**2)
      IF(.NOT.(RESID.LT.1.0D-7)) GO TO 35
         EXZSMN = .5/PI
      GO TO 36
   35 CONTINUE
```

```
         EXZSMN = SIGN*DIM*(NUMER/DENOM)
   36 CONTINUE
      RETURN
      END
C SUBR*****************************************************************
$EJECT
      SUBROUTINE EXZSH3(ZETAD,LERES,LPROVL,TOVD,EXZE3)
C**********************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE FOURIER TRANSFORM *
C OF THE SPATIAL SHIFT OF THE Z-COMPONENT OF THE ELECTRIC FIELD FOR  *
C THE EVEN MCCE WHERE THE SHIFT FUNCTION IS A COSINE FUNTION.         *
C*****VARIABLE DEFINITIONS*********************************************
C EXZE3 = THE FOURIER TRANSFORM OF THE COS SHIFT FUNCTION OF THE Z-   *
C         COMPONENT OF THE ELECTRIC FIELD FOR THE EVEN MODE, THE      *
C         RETURNED VALUE.                                             *
C*****VARIABLE DECLARATION*********************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      REAL*8 LERES,LPROVL,LEOVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      LEOVD=LERES*LPROVL/DOVL
C CALCULATE EXZESH, THE EVEN MODE Z TRANSFORM FACTOR
      EXZE3 = DCOS(ZETAD*.5*(LEOVD+TOVD))
      RETURN
      END
C SUBR*****************************************************************
$EJECT
      SUBROUTINE EXZSH4(ZETAD,LORES,LPROVL,TOVD,EXZO4)
C**********************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE FOURIER TRANSFORM *
C OF THE SPATIAL SHIFT OF THE Z-COMPONENT OF THE ELECTRIC FIELD FOR  *
C THE ODD MODE WHERE THE SHIFT FUNCTION IS A SINE FUNTION.            *
C*****VARIABLE DEFINITIONS*********************************************
C EXZO4 = THE FOURIER TRANSFORM OF THE SIN SHIFT FUNCTION OF THE Z-   *
C         COMPONENT OF THE ELECTRIC FIELD FOR THE ODD MODE, THE       *
C         RETURNED VALUE.                                             *
C*****VARIABLE DECLARATION*********************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      REAL*8 LORES,LPROVL,LOOVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      LOOVD=LORES*LPROVL/DOVL
C CALCULATE EXZOSH THE ODD MODE Z TRANSFORM FACTOR
      EXZO4 =  DSIN(ZETAD*.5*(LOOVD+TOVD))
      RETURN
      END
C SUBR*****************************************************************
$EJECT
```

```
      SUBROUTINE EXZSH5(ZETAD,LERES,LPROVL,TCVD,EXZE5)
C***********************************************************************
C      THIS SUBROUTINE IS DESIGNED TO CALCULATE THE FOURIER TRANSFORM  *
C OF THE SPATIAL SHIFT OF THE Z-COMPONENT OF THE ELECTRIC FIELD FOR    *
C THE EVEN MODE WHERE THE SHIFT FUNCTION IS A SINE FUNTION.            *
C*****VARIABLE DEFINITIONS*********************************************
C EXZE5 = THE FOURIER TRANSFORM OF THE SIN SHIFT FUNCTION OF THE Z-    *
C          COMPONENT OF THE ELECTRIC FIELD FOR THE EVEN MODE, THE      *
C          RETURNED VALUE.                                             *
C*****VARIABLE DECLARATION*********************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      REAL*8 LERES,LPROVL,LEOVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      LEOVD=LERES*LPROVL/DOVL
C CALCULATE  EXZESH THE ODD MODE Z TRANSFORM FACTOR
      EXZE5=  DSIN(ZETAD*.5*(LEOVD+TOVD))
      RETURN
      END
C SUBR******************************************************************
$EJECT
      SUBROUTINE EXZSH6(ZETAD,LORES,LPROVL,TOVD,EXZO6)
C***********************************************************************
C      THIS SUBROUTINE IS DESIGNED TO CALCULATE THE FOURIER TRANSFORM  *
C OF THE SPATIAL SHIFT OF THE Z-COMPONENT OF THE ELECTRIC FIELD FOR    *
C THE ODD MODE WHERE THE SHIFT FUNCTION IS A COSINE FUNTION.           *
C*****VARIABLE DEFINITIONS*********************************************
C EXZO6 = THE FOURIER TRANSFORM OF THE COS SHIFT FUNCTION OF THE Z-    *
C          COMPONENT OF THE ELECTRIC FIELD FOR THE ODD MODE, THE       *
C          RETURNED VALUE.                                             *
C*****VARIABLE DECLARATION*********************************************
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      REAL*8 LORES,LPROVL,LOCVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      LOOVD=LORES*LPROVL/DOVL
C CALCULATE EXZOSH, THE ODD MODE Z TRANSFORM FACTOR
      EXZO6= DCOS(ZETAD*.5*(LOOVD+TOVD))
      RETURN
      END
C SUBR******************************************************************
C  A. IDENTIFICATION:                                                   *
C      TITLE:        DETERMINANT OF A REAL MATRIX                       *
C      CATEGORY:     DETERMINANTS                                       *
C      PROGRAMMER:   JEAN BOW                                           *
C      DATE:         FEBRUARY 1965, CONVERTED TO SYSTEM/360, JANUARY 1967 *
C                    UPDATED BY P.J. COLLINS JANUARY 1984               *
C  B. PURPOSE:                                                          *
```

```
C          THIS SUBROUTINE COMPUTES THE DETERMINANT OF A MATRIX OF REAL   *
C          NUMBERS BY GAUSS' METHOD OF ELIMINATION WITH ROW PIVOTING.     *
C     C. USAGE:                                                           *
C        1. CALLING SEQUENCE:                                             *
C           CALL DTERM(N,A,DET,M)                                         *
C        2. PARAMETERS:                                                   *
C           N - ORDER OF MATRIX A. (INTEGER)                              *
C           A - MATRIX, THE DETERMINANT OF WHICH IS TO BE COMPUTED.(REAL*8)
C           DET - DETERMINANT OF MATRIX A. (REAL*8)                       *
C           M - ROW DIMENSION OF MATRIX A. (THE INTEGER APPEARING IN USER'S
C               DIMENSION STATEMENT.)                                     *
C     D. CAUTIONS TO USER:                                                *
C        IN THE CALLING PROGRAM, "A" MUST APPEAR IN A DIMENSION STATEMENT.*
C        MATRIX A IS DESTROYED.                                           *
C     E. REFERENCES:                                                      *
C        1. KUNZ, K. S., "NUMERICAL ANALYSIS", MCGRAW-HILL, NEW YORK, 1957*
C        2. FADDEVA, V. A., "COMPUTATIONAL METHODS OF LINEAR ALGEBRA",    *
C           TRANSLATED BY CURTIS D. BENSTER, DOVER PUBLICATIONS, INC.,    *
C           NEW YORK, 1959.                                               *
C*************************************************************************
$EJECT
      SUBROUTINE DTERM(N,A,D,M)
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      DIMENSION A(M,M)
      DD=1.0D0
   10 DO 34 L=1,N
         KP=C
         Z=0.0
         DO 12 K=L,N
            IF(Z-DABS(A(K,L)))11,12,12
   11       Z=DABS(A(K,L))
            KP=K
   12    CONTINUE
         IF(L-KP)13,20,20
   13    DO 14 J=L,N
            Z=A(L,J)
            A(L,J)=A(KP,J)
   14    A(KP,J)=Z
         DD=-DD
   20    IF(L-N)31,40,40
   31    LP1=L+1
         DO 34 K=LP1,N
         IF(A(K,L))32,34,32
   32    RATIO=A(K,L)/A(L,L)
         DO 33 J=LP1,N
   33    A(K,J)=A(K,J)-RATIO*A(L,J)
   34 CONTINUE
   40 DO 41 K = 1,N
```

```
   41  DD=DD*A(K,K)
       D=DD
       RETURN
       END
$ENTRY
.0677333    .0719667    .0762       .0804333    .0846667    .0889       .0931333
.0973667    .1016
1.          1.          1.
4.5         3.5         4.0
1.0         0.5         0.25        0.2         0.1
0.2         0.5         1.0         2.0         5.0         10.0
$END
```

LIST OF REFERENCES

1. Tischer, F.J., "Transmission Media for the Design of Millimeter-Wave Circuitry", 8th European Microwave Conference 1978, Workshop on Millimeter Wave Circuits.

2. Meier, P.J., "Integrated Fin-Line Millimeter Components", IEEE Trans. MTT, v. 22, pp. 1209-1216, December 1974.

3. Chang, C., and Itoh, T., "Spectral Domain Analysis of Dominant and Higher Order Modes in Fin-Lines", 1979 IEEE MTT-S Int. Microwave Symp., (Orlando, Florida), pp. 344-346.

4. Knorr, J.B., and Shayda, P.M., "Millimeter-Wave Fin-Line Characteristics", IEEE Trans. MTT, v. 28, pp. 737-743, July 1980.

5. Schmidt, L.P., and Itoh, T., "Spectral Domain Analysis of Dominant and Higher Order Modes in Fin-Lines", IEEE Trans MTT, v. 28, pp. 981-985, September 1980.

6. El-Sherbiny, A.M.A., "Exact Analysis of Shielded Microstrip Lines and Bilateral Fin-Lines", IEEE Trans. MTT, v. 29, pp. 669-675, July 1981.

7. Knorr, J.B., "Equivalent Reactance of a Shorting Septum in a Fin-Line: Theory and Experiment", IEEE Trans. MTT, v. 29, pp. 1196-1202, November 1981.

8. Mirshekar-Syahkal, D., and Davies, J.B., "Accurate Analysis of Couples Strip-Finline Structure for Phase Constant, Characteristic Impedance, Dielectric and Conductor Losses", IEEE Trans. MTT, v. 30, pp. 906-910, June 1982.

9. Helard, M., Citerine, J., Picon, O., and Fouad Hanna, V., "Solution of Fin-Line Discontinuities Through the Identification of Its First Four Higher Order Modes", IEEE MITT-S Digest, pp. 387-389, 1983.

10. Sharma, A.K., and Hoefer, W.J.R., "Empirical Expressions for Fin-Line Design", IEEE Trans. MTT, v. 31, pp. 350-355, April 1983.

11. Sharma, A.K., and Hoefer, W.J.R., "Propagation in Coupled Unilateral and Bilateral Finlines", IEEE Trans. MTT, v. 31, pp. 498-501, June 1983.

12. Harrington, R.F., Field Computation by Moment Methods, Macmillan, 1968.

13. Demidovich, B.P., and Maron, I.A., Computational Mathematics, Mir, 1976.

14. Brown, R.G., Sharpe, R.A., Hughes, W.L., and Post, R.E., Lines, Waves, and Antennas, Wiley, 1973.

15. Miller, G.S., An Experimental Investigation of Several Fin-Line Discontinuities, M.S. Thesis, Naval Postgraduate School, Monterey, California, 1980.

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